A DELAYED REPAIR MODEL

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Abstract

In this work, we consider a system that operates during a working day and it is subjected to failures. When a failure occurs, repair begins immediately. It is proposed a new repair policy: when it is reached a certain time of the working day, channel repair is closed and repair postponed until the end of the working day. Under particular conditions, we show that this repair policy would be worthwhile.

1 The model

We consider a one-component system and one repairman. Systems unit is operating in a working day and it is subjected to failures. Working day begins at instant 0 and it ends at instant T. Company, systems owner, obtains a reward when unit is operating and it suffers a high cost when it is being repaired during working day. This high cost is due to the immediate accessibility to an expert repair.

Company managers propose a new repair policy and they call it delayed repair policy: when it is reached a certain time T^*, being $T^* < T$, channel repair will be closed and repair postponed until the end of the working day. During the period of rest, there is a maintenance equipment that makes the repair without additional costs.

However, delayed repair has got some disadvantages: mean operating time when repair has been postponed is less than mean operating time when repair has not been postponed. Besides, if the repair is postponed and a failure occurs again, the unit will be down and production will stop until next workday.

We make the following additional assumptions.

1.1 Assumptions

1. At the beginning of the working day a new system is ready and unit starts operating at instant 0.

2. Let X_i, $i = 1, 2$, be the operating time of the unit when $i-1$ repairs have been postponed. We assume that each X_i is governed by an exponential distribution of parameter λ_i, $i = 1, 2$. We will also assume $\lambda_2 > \lambda_1$, i.e., mean operating time when repair has been postponed is less than mean operating time when repair has not been postponed.

3. Let Y be the repairing time of the unit. We assume Y is governed by an exponential distribution of parameter μ. Repair of the unit is as good as new. Transfer time to the channel repair is negligible.

4. X_1, X_2 and Y are independent random variables.

5. Company obtains a reward of A_0 monetary units (m.u.) per unit time when unit is operating and it is produced a cost of A_r m.u. per unit time when unit is under repair. Repair during the period of rest has not additional costs.

1.2 Transition Probabilities

For $0 \leq t \leq T^*$

- Let $P_o(t)$ be the probability that unit is operating at instant t and $P_r(t)$ be the probability that unit is under repair at instant t. We note by $E(W_o)$ and $E(W_r)$ the mean operating time and the mean repairing time in $[0, T^*]$, respectively.

For $T^* < t \leq T$,

- Let $P_{io}(t)$, $i = 1, 2$, be the probability that unit is operating at time t and it has been postponed $i-1$ repairs conditioned to unit being operating at T^*. We note by $E(W_{io})$, $i = 1, 2$, the mean time the system stays in state io in $(T^*, T]$.
- Let $P_{do}(t)$ be the probability unit is down at instant t conditioned to unit being operating at T^*. We note by $E(W_{do})$ the mean time the system stays in that state.
- Analogously, let $P_{rr}(t), P_{1r}(t), P_{2r}(t), P_{dr}(t)$ be the probability that unit is being repaired, being operating when repair has not been postponed, being operating when repair has been postponed and being down, respectively, conditioned to unit is under repair at time T^*. We note by $E(W_{rr}), E(W_{1r}), E(W_{2r}), E(W_{dr})$ the mean time in those states in $(T^*, T]$.

Because of the exponential distribution of X_1 and Y and by the initial condition $P_o(0) = 1$, it is obtained

$$\begin{aligned} P_o(t) &= \frac{\mu}{\lambda_1 + \mu} + \frac{\lambda_1}{\lambda_1 + \mu} e^{-(\lambda_1 + \mu)t}, \quad 0 \leq t \leq T^* \\ P_r(t) &= \frac{\lambda_1}{\lambda_1 + \mu} - \frac{\lambda_1}{\lambda_1 + \mu} e^{-(\lambda_1 + \mu)t}, \quad 0 \leq t \leq T^* \end{aligned}$$

and

$$\begin{aligned} E(W_0) &= \int_{t=0}^{T^*} P_o(t)dt = \frac{\mu T^*}{\lambda_1 + \mu} + \frac{\lambda_1}{(\lambda_1 + \mu)^2} \left[1 - e^{-(\lambda_1 + \mu)T^*}\right] \\ E(W_r) &= \int_{t=0}^{T^*} P_r(t)dt = \frac{\lambda_1 T^*}{\lambda_1 + \mu} - \frac{\lambda_1}{(\lambda_1 + \mu)^2} \left[1 - e^{-(\lambda_1 + \mu)T^*}\right] \end{aligned}$$

Let

$$F_i(x) = 1 - e^{-\lambda_i x}, \quad \forall x \geq 0, \quad i = 1, 2$$

and

$$F_r(x) = 1 - e^{-\mu x}, \quad \forall x \geq 0$$

Calculating, we obtain the following intuitive equalities

$$E(W_{1o}) = \int_{t=T^*}^{T} \bar{F}_1(t - T^*)dt = \frac{1}{\lambda_1} F_1(T - T^*) \tag{1}$$

$$E(W_{2o}) = \int_{t=T^*}^{T} \int_{u=T^*}^{t} f_1(u - T^*)\bar{F}_2(t - u) = \frac{1}{\lambda_2} F_{1+2}(T - T^*) \tag{2}$$

$$E(W_{rr}) = \int_{t=T^*}^{T} \bar{F}_r(t - T^*)dt = \frac{1}{\mu} F_r(T - T^*) \tag{3}$$

$$E(W_{1r}) = \int_{t=T^*}^{T} \int_{u=T^*}^{t} f_r(u - T^*)\bar{F}_1(t - u) = \frac{1}{\lambda_1} F_{1+r}(T - T^*) \tag{4}$$

$$E(W_{2r}) = \int_{t=T^*}^{T} \int_{u=T^*}^{t} f_r(u-T^*) \int_{v=u}^{t} f_1(v-u)\bar{F}_2(t-v) = \frac{1}{\lambda_2} F_{r+1+2}(T-T^*) \quad (5)$$

being F_{1+2} the distribution function (d.f.) for the sum of two random variables with d.f. F_1 and F_2, respectively, F_{r+1} the one for the sum of two random variables with d.f. F_1 and F_r, resp. and F_{1+2+r} the one for the sum of three random variables with d.f. F_1, F_2 and F_r, resp.

2 Profit Function

We note by $R(T^*)$ the expected reward obtained in a working day

$$\begin{aligned} R(T^*) &= A_0 E(W_o) - A_r E(W_r) + P_o(T^*)[A_0(E(W_{1o}) + E(W_{2o}))] + \\ &+ P_r(T^*)[-A_r E(W_{rr}) + A_0 E(W_{1r}) + A_0 E(W_{2r})] \end{aligned} \quad (6)$$

Derivating in (6) in respect of T^*, we have

$$\begin{aligned} \frac{dR(T^*)}{dT^*} &= P_o(T^*)\left[A_0 + A_0 \frac{dE(W_{1o})}{dT^*} + A_0 \frac{dE(W_{2o})}{dT^*}\right] + \\ &+ P_r(T^*)\left[-A_r - A_r \frac{dE(W_{rr})}{dT^*} + A_0 \frac{dE(W_{1r})}{dT^*} + A_0 \frac{dE(W_{2r})}{dT^*}\right] + \\ &+ \frac{dP_o(T^*)}{dT^*}[A_0 E(W_{1o}) + A_0 E(W_{2o}) + A_r E(W_{rr}) - A_0 E(W_{1r}) - A_0 E(W_{2r})] \end{aligned}$$

If $T^* = T$, it is obtained

$$E(W_{1o}) = E(W_{2o}) = E(W_{rr}) = E(W_{1r}) = E(W_{2r}) = 0$$

and, using (1), (2), (3), (4) and (5),

$$\left.\frac{dE(W_{1o})}{dT^*}\right|_{T=T^*} = \left.\frac{dE(W_{rr})}{dT^*}\right|_{T=T^*} = -1$$

$$\left.\frac{dE(W_{2o})}{dT^*}\right|_{T=T^*} = \left.\frac{dE(W_{1r})}{dT^*}\right|_{T=T^*} = \left.\frac{dE(W_{2r})}{dT^*}\right|_{T=T^*} = 0$$

so

$$\left.\frac{dR(T^*)}{dT^*}\right|_{T^*=T} = 0$$

therefore $T^* = T$ is a local extreme.

Calculating, it is obtained that

$$\left.\frac{d^2E(W_{1o})}{dT^{*2}}\right|_{T=T^*} = -\lambda_1, \quad \left.\frac{d^2E(W_{2o})}{dT^{*2}}\right|_{T=T^*} = \lambda_1$$

$$\left.\frac{d^2E(W_{rr})}{dT^{*2}}\right|_{T=T^*} = -\mu, \quad \left.\frac{d^2E(W_{1r})}{dT^{*2}}\right|_{T=T^*} = \mu, \quad \left.\frac{d^2E(W_{2r})}{dT^{*2}}\right|_{T=T^*} = 0$$

and

$$\left.\frac{d^2R(T^*)}{dT^*}\right|_{T=T^*} = (A_0 + A_r)\left(P_r(T)f_r(0) - \left.\frac{dP_o(T^*)}{dT^*}\right|_{T=T^*}\right)$$

and $P_o(t)$ is decreasing, therefore

$$\left.\frac{d^2R(T^*)}{dT^*}\right|_{T=T^*} > 0$$

thus $T^* = T$ is a minimum for $R(T^*)$. So we can conclude that a delayed repair policy is worthwhile.

References

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