

Evaluation of reliability bounds by genetic algorithms

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Abstract

In the present article we deal with the problem of developing a systematic procedure for evaluating the general reliability bounds published recently by Fu and Koutras (1995). More specifically, we prove that, the identification of the optimal bounds can be achieved by transforming the set-theoretic and probabilistic conditions associated with them to an equivalent set covering problem (*SCP*). As a consequence, genetic algorithms for the *SCP* can be exploited to derive very tough approximation intervals for a general system's reliability at very competitive computer times as compared to the respective exact reliability evaluation algorithms. A brief discussion for future research along these lines is also included.

1. Introduction

In the present article we consider binary reliability structures, that is systems consisting of components that can only be in one of two states, operational (working, on, up) or failed (not working, off, down). Let $I=\{1,2,\dots,n\}$ be the set of all components of the system and denote by z_i , $i \in I$ the state of component i at a fixed instance, that is

$$z_i = \begin{cases} 1 & \text{if component } i \text{ is working} \\ 0 & \text{if component } i \text{ is not working.} \end{cases}$$

The state of the system is completely determined by the state of its components, and the dependence of the system state on the element states is usually expressed through the so-called structure function $\varphi(z_1, z_2, \dots, z_n)$ which takes on the value 1 if the system is up and the value 0 if the system is down. An alternative method to describe a reliability structure is through the family of minimal cut sets

$$C = \{C_1, C_2, C_3, \dots, C_N\}$$

or the family of minimal path sets

$$P = \{P_1, P_2, P_3, \dots, P_M\}.$$

In spite of the fact that typically the reliability of the system can be written as

$$R_n = E[\varphi(Z_1, Z_2, \dots, Z_n)]$$

where Z_i is a binary random variable associated with the i -th component's state at prespecified instance, the evaluation of R_n by the aid of this formula is, in general, computationally intractable.

From now on we assume that the components of the system work independently, and denote by $p_i=E(Z_i)$, $q_i=1-p_i$, $i \in I$ their survival and failure probabilities respectively. One of the earliest approaches towards constructing simple reliability bounds for coherent structures with independent components, should be attributed to Esary and Proschan (1963) who proved that

$$L_{EP} = \prod_{j=1}^N (1 - \prod_{i \in C_j} (1 - p_i)) \leq R_n \leq 1 - \prod_{j=1}^M (1 - \prod_{i \in P_j} p_i) = U_{EP}$$

Unfortunately, the Esary and Proschan bounds cannot be effectively used for approximating system reliability from both sides (above and below); as numerical experimentation indicates, L_{EP} yields good approximations for high reliability structures while U_{EP} performs well only for low reliability structures. Motivated by this observation, Fu and Koutras (1995) established recently two

classes of reliability bounds which can be effectively coupled with the Esary and Proschan bounds to construct very tight intervals estimates; see also Boutsikas and Koutras (2000) for a further generalization. Their bounds have the form

$$L_{FK} = 1 - \prod_{j=1}^M [1 - (\prod_{i \in K_j} q_i)(\prod_{i \in P_j} p_i)] \leq R_n \leq \prod_{j=1}^N [1 - (\prod_{i \in L_j} p_i)(\prod_{i \in C_j} q_i)] = U_{FK}$$

where $L_j, K_j \subseteq I$ are appropriate index sets satisfying specific conditions. Although there always exist such index sets, the authors have not provided any standard procedure leading to the determination of them.

2. The Algorithm

The optimum choice of L_j (in the sense that the respective upper bound U_{FK} shifts as close to the lower bound L_{EP} as possible) is achieved by minimizing the sum

$$f(\mathbf{x}) = \sum_{k \in C_j} (-\ln p_k) x_k = \sum_{k \in C_j} c_k x_k \quad (2.1)$$

where $c_k = -\ln p_k$, $k=1,2,\dots,n$. In order to establish an algorithmic procedure yielding lower and upper reliability bounds for a general structure, we introduce a binary matrix $A = (\alpha_{ik})_{N \times n}$ by

$$\alpha_{ik} = \begin{cases} 1 & \text{if component } k \text{ belongs to minimal cut set } C_i \\ 0 & \text{if component } k \text{ does not belong to minimal cut set } C_i. \end{cases} \quad (2.2)$$

Then, the following algorithm can be exploited for deducing both the Esary and Proschan lower bounds L_{EP} and the bound U_{FK} for a coherent structure whose family of minimal cut sets (1.1) has been given.

Step 1. Set $L_{EP}=U_{FK}=1$ (initial values) and evaluate c_k and α_{ik} by (2.1) and (2.1) respectively.

Step 2. Repeat steps 3-6 for $j=1,2,\dots,N$

Step 3. Set $L_{EP} = L_{EP} * (1 - \prod_{i \in C_j} q_i)$.

Step 4. Compute the auxiliary index sets L_j^* defined by

$$L_1^* = \emptyset, \quad L_j^* = \{i : C_i \cap C_j \neq \emptyset, 1 \leq i \leq j\}, \quad j = 2,3,\dots,N.$$

If $L_j^* = \emptyset$ then set $U_{FK} = U_{FK} * (1 - \prod_{i \in C_j} q_i)$ and return to step 3 (for the next j).

If $L_j^* \neq \emptyset$ then perform steps 5-6.

Step 5. Solve the weighted set covering problem (SCP):

$$\begin{aligned} & \text{Minimize } \sum_{t \in T} c_t x_t \\ & \text{Subject to } \sum_{t \in T} \alpha_{vt} x_t \geq 1, \quad \text{for all } v \in V, \quad x_t \in \{0,1\}, \quad \text{for all } t \in T \end{aligned}$$

where

$V = L_j^*$ the set of rows to be covered, $T = C'_j$ the set of columns for covering the rows

Step 6. If $\{x_t, t \in T\}$ is the solution obtained in step 5, compute L_j by $L_j = \{t \in T : x_t = 1\}$

and set $U_{FK} = U_{FK} [\prod_{j=1}^N [1 - (\prod_{i \in L_j} p_i)(\prod_{i \in C_j} q_i)]]$. Then return to step 3 (for the next j).

After having completed steps 1-6, the variable L_{EP} and U_{FK} contain the Esary and Proschan lower bound and the optimal Fu and Koutras upper bound for system's reliability respectively.

3. Genetic algorithms for the *SCP* and numerical results

In order to arrive at the computation of U_{FK} , through the algorithm described in Section 2, we have to solve a series of set covering instances. Clearly, pursuing the exact solution of each one, would result to prohibitive execution times. As an alternative to this, we may use a Genetic Algorithm (*GA*) to approximate the solutions for each instance (for a related discussion on meta-heuristics efficiency, see Hordijk and Stadler (1998) or Angel and Zissimopoulos (2000)), thereof obtaining an upper bound of the exact value of U_{FK} . In a recent paper, Beasley and Chu (1996) introduced a *GA* for *SCP*, which is capable of producing high quality solutions. In our numerical calculations we used the same algorithm with some modifications (see Tsitmidelis, Zissimopoulos, Koutras (2001)).

The first step before applying a *GA*, is to decide on a suitable representation scheme for encoding the population. For the *SCP*, the usual 0-1 binary string representation is an obvious choice, since it can describe directly the underlying 0-1 integer variables $x_t, t \in T$. Thus, each individual is encoded as a n -bit binary string, where n is the number of columns in the *SCP*. A value of 1 at the i -th bit implies that column i belongs to the solution. The *GA* is launched by producing N random individuals as the initial population. Next, the fitness of each individual i is calculated as $\sum_{j=1}^n c_j b_{ij}$, where b_{ij} is the value of j -th bit (column) in the string corresponding to the i -th chromosome (child) and c_j is the cost of bit (column) j .

To test the efficacy of our algorithm, two types of problems were treated. In the first we considered the class of consecutively connected systems and performed repeated calculations selecting several system sizes n , component reliabilities p_i and transmission capabilities k_i . In the second we generated the system structure at random by selecting the cut sets among the power set of I so that the resulting structure consists a coherent (monotone) system. In Table 1, besides the Esary and Proschan lower bound L_{EP} and the Fu and Koutras upper bound U_{FK} , we have also included (to make the numerical comparisons more easy) the reliability estimate $R_E = (U_{FK} + L_{EP})/2$ and the exact reliability value R . A direct inspection of the relative error column (which was computed as $(U_{FK} - L_{EP})/2L_{EP}$) reveals that, our algorithm leads to quite tight intervals for system's reliability at very reasonable computer times.

Table 1: Genetic algorithm-based reliability bounds for consecutively connected systems

P	N	n	L_{EP}	U_{FK}	R_E	R	Computer time (sec)	Relative error
0.85-0.95	26	50	0,8443318	0,8449217	0,8446268	0,8443938	22,07	3,49E-04
0.85-0.95	40	100	0,8476565	0,8476698	0,8476632	0,8476566	102,93	7,85E-06
0.80-0.90	52	100	0,7854174	0,7864784	0,7859479	0,7854555	129,73	6,75E-04
0.60-0.70	80	150	0,5329428	0,5400552	0,5364990	0,5332648	417,89	6,67E-03
0.80-0.90	80	150	0,7905573	0,7908107	0,7906840	0,7905586	404,61	1,60E-04
0.90-0.95	80	150	0,8975495	0,8975580	0,8975538	0,8975495	432,63	4,74E-06
0.60-0.70	36	200	0,5457909	0,5459874	0,5458892	0,5458336	174,19	1,80E-04
0.80-0.90	36	200	0,7915937	0,7915940	0,7915939	0,7915937	176,25	1,89E-07
0.90-0.95	36	200	0,8976591	0,8976591	0,8976591	0,8976591	176,41	0,00E+00
0.60-0.70	99	200	0,5454149	0,5455745	0,5454947	0,5454165	855,13	1,46E-04
0.80-0.90	99	200	0,7918385	0,7918386	0,7918386	0,7918385	867,25	6,31E-08
0.60-0.70	252	500	0,5449945	0,5453491	0,5451718	0,5449953	13751,72	3,25E-04
0.80-0.90	252	500	0,7917534	0,7917546	0,7917540	0,7917534	19961,66	7,58E-07

The same remarks apply for Table 2, where the system structure was randomly generated; note though that for these cases the exact reliability value cannot be evaluated unless a complete search is performed over the 2^n configurations resulting from the states of the n components (something that would require extraordinary computer times)

Table 2: Genetic algorithm-based reliability bounds for random systems

P	N	n	L_{EP}	U_{FK}	R_E	Computer time(sec)	Relative error
0.80-0.90	98	50	0,9626803	0,9637253	0,9632028	36,83	5,43E-04
0.60-0.70	50	100	0,9952515	0,9977028	0,9964772	55,81	1,23E-03
0.80-0.90	50	100	0,9971187	0,9971570	0,9971379	38,45	1,92E-05
0.80-0.90	49	100	0,9422784	0,9422919	0,9422852	19,26	7,16E-06
0.80-0.90	100	200	0,9997982	0,9998102	0,9998042	149,61	6,00E-06
0.80-0.90	100	200	0,9479110	0,9479530	0,9479320	140,08	2,22E-05
0.50-0.60	100	500	0,9999994	0,9999995	0,9999995	833,87	5,00E-08
0.50-0.60	100	500	0,9991869	0,9994975	0,9993422	578,79	1,55E-04
0.80-0.90	100	500	0,9980323	0,9980953	0,9980638	159,03	3,16E-05
0.80-0.90	250	500	0,9630716	0,9630755	0,9630736	342,95	2,02E-06

Two issues seem to be crucial and need further investigation. The first pertains to whether one could exploit the interrelations between the cut sets of the system to avoid repeating the genetic algorithm from scratch each time. The second is related to the problem of establishing an efficient procedure to arrive the most efficient order of the cut sets so that better upper and lower bounds of system's reliability are obtained. To achieve this goal a travelling salesman model (*TSP*) can be set up that could be subsequently solved by exploiting a genetic algorithm. It is clear that, succeeding in the first task would enable us to reduce considerably the required computational time, while an advancement towards the second goal would lead to higher quality estimates of system's reliability.

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