Some questions about symplectic and multi-symplectic discretizations

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The talk will discuss two questions in geometric integration. The first question is: what is the appropriate geometric integrator for simulating overturning and breaking water waves? The widely used governing equations for modelling water waves are Hamiltonian, and therefore one would expect that symplectic integrators would be appropriate for time integration. When the surface is a graph, the symplectic structure is canonical. But for general surfaces, for example breaking waves, one needs a coordinate-free Hamiltonian formulation, and this was first proposed by Benjamin & Olver (1982). This Hamiltonian structure is no longer canonical, and new ideas from geometric integration are needed. Mathematically, the problem reduces to tracking a two-dimensional surface in 3−space, and has similarites with curvature-driven surface flow.

The second question is: how does one combine multi-symplectic structures on a manifold with "difference forms" or "discrete differential forms"? It has recently been shown that on any smooth Riemannian manifold, there is a "canonical multi-symplectic structure" on the total exterior algebra bundle. This structure gives a coordinate-free formulation of a large class of nonlinear elliptic PDEs (or hyperbolic PDEs if the manifold is pseudo-Riemannian). The canonical multi-symplectic structure is characterized in terms of differential forms on the *base manifold*, rather than the fibre. Therefore a natural way to discretize this structure is to use the theory of discrete differential forms or difference forms. The talk will discuss progress with developing this idea.