Explicit Magnus expansions for nonlinear equations

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Nowadays the Magnus expansion constitutes a widespread tool to construct approximate solutions of time dependent systems of linear differential equations. From a numerical point of view, the resulting algorithms have proved to be competitive with other, more conventional numerical schemes with respect to accuracy and computational effort, whilst preserving the geometric structure of the original system.

In this talk we develop and analyse new explicit Magnus expansions for the nonlinear equation

$$Y' = A(t, Y)Y, \qquad Y(0) = Y_0 \in \mathcal{G},$$

where $\mathcal{G}$ is a matrix Lie group. In particular, integration methods up to order four are presented in terms of integrals which can be either exactly evaluated or replaced by conveniently adapted quadrature rules. The structure of the algorithm allows to use to change the step size and even the order along the integration process, thus improving its efficiency. Several examples are considered, including isospectral flows and hihgly-oscillatory nonlinear differential equations.

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