Eulerian and Semi-Lagrangian exponential integrators for convection dominated problems

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Exponential integrators have been proposed and studied in the literature especially when applied to parabolic PDEs which in their semidiscretized form present a dominating linear stiff part and a nonlinear non-stiff part. Typically in the exponential integrator the exact integration of the linear stiff part is performed.

We propose here a new class of integration methods particularly suited for convection diffusion problems with dominating convection. These methods are exponential integrators and their peculiarity is that they allow for the computation of exponentials of the linearized convection term.

The main reason for developing this type of methods is that as it turns out they can be applied to the numerical integration of the considered PDEs in a semi-Lagrangian fashion. The main challenge in the numerical approximation of convection dominated phenomena is to avoid the occurrence of spurious oscillations in the numerical solution, (numerical dispersion), without adding diffusion. This task is achieved nicely by semi-Lagrangian methods. In these methods linear convective terms are integrated *exactly* by computing first the characteristics corresponding to the gridponts of the adopted discretization, and then producing the numerical approximation via a simple although expensive interpolation procedure.