

On an isospectral Lie–Poisson flow and numerical computation of faithful Lie-algebra representations

^aA. Iserles

^a Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences,
University of Cambridge, Wilberforce Rd, Cambridge CB3 0WA, UK.

Our point of departure is the matrix flow $X' = [N, X^2]$, $X(0) \in \text{Sym}(n)$, $N \in \mathfrak{so}(n)$, which is subject to two distinct group actions, by similarity and by congruence. We demonstrate that it is isomorphic to an integrable Lie–Poisson flow and explore its invariants and Casimirs.

As is well known in geometric integration, there are important advantages in solving Lie–Poisson systems in the dual to underlying Lie algebra. However, in our case the Lie algebra is *a priori* unknown: we are given just its structure constants. Hence the task of determining a faithful representation: essentially, rendering the Ado theorem constructively in our special case. Although there exists a general symbolic algorithm for the computation of faithful representations, due to Willem de Graaf, it leads in our case to excessively large matrices which render computation in the dual very expensive indeed. Instead, we treat the task in hand as a problem in numerical linear algebra and present a very effective algorithm based on complex-valued Cholesky factorization.

Using a different approach, we demonstrate that the underlying Lie algebra is isomorphic to the symplectic algebra.

This work has been done in collaboration with Tony Bloch, Jerry Marsden and Tudor Ratiu.