Lie Group and Elementary Differential Variational Integrators with Applications to Full Body Problems

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The study of attitude dynamics and control has been motivated by applications in robotics, space flight systems, ground-based pointing and tracking systems. In spite of the extensive literature on these problems, there has been little attention in the engineering literature given to efficient and accurate computational approaches that respect the geometry underlying these problems.

We introduce a variational integrator that performs computations at the level of the Lie algebra, and updates the solution using the matrix exponential. Consequently, the attitude automatically evolves on the rotation group embedded in the space of matrices, without the need for reprojection onto the manifold, or the use of constraints.

We will also discuss the construction of elementary differential variational integrators, which are obtained by considering discrete Lagrangians arising from quadrature rules that incorporate derivative information. These schemes can be interpreted as generalizations of symplectic Runge–Kutta methods, which utilize derivatives of the Hamiltonian vector field in propagating the numerical solution. This provides a general framework for constructing variational integrators that overcome the barrier order associated with traditional Runge–Kutta schemes, and provides a systematic method of constructing symplectic methods with s stages, but with order of accuracy greater than $2s$.

This work is in collaboration with Taeyoung Lee and N. Harris McClamroch.