Fourier analysis on groups applied to spectral element discretizations of PDEs

H. Munthe-Kaas[a]

[a] Department of Informatics, University of Bergen, N-5020 Bergen, Norway

In this talk we will discuss some applications of Fourier analysis on groups in the solution of PDEs. Equivariant discretizations yields structured matrices that can be block diagonalized using the Generalized Fourier Transform (GFT). An example being equivariant discretisations for PDEs on spheres, where the use of GFT over the icosahedral symmetry group yields fast algorithms for computing matrix exponentials, which is a basic computation for Lie group integrators.

This leads to the important question of how to construct spectral element discretizations based on triangular subdivisions. By taking the quotient of a compact abelian group with a finite group of automorphisms, we obtain families of multivariate generalizations of Chebyshev polynomials on certain non-separable domains. This yields an approximation theory for triangular domains suitable for the construction of spectral element bases. We will show that these bases share most of the beautiful properties of classical Chebyshev expansions, such as near-optimal Lebesgue constants for the interpolation error and the existence of fast transforms for expansions and spectral differentiation.