On Geometric Integrators for the Nonlinear Schrödinger equation

B. Owren[a]

[a] Department of Mathematical Sciences, NTNU, N-7491 Trondheim, Norway

The nonlinear Schrödinger equation

$$iu_t + u_{xx} = \lambda |u|^2 u$$

with periodic boundary conditions is considered. A number of geometric integrators have been developed for this system, many of them are summarized in [Islas et al., Geometric Integrators for the nonlinear Schrödinger equation, J. Comp. Phys. 173 (2001)]. One may use a multisymplectic formulation and use multisymplectic discretisations as e.g. the box schemes. Or one may use various time discretisations obtained from the Ablowitz-Ladik system which is a completely integrable ODE system. These integrators seem to work well for a moderate number of spatial degrees of freedom, but in the limit, several difficulties occur. There are nonlinear systems to be solved in each time step, and as their size grows, the convergence deteriorates substantially. Unless very strong convergence criteria are imposed, one typically observes numerical resonance for moderate values of the time step. More recently, a conservative scheme has been developed in [Besse, A relaxation scheme for the nonlinear Schrödinger equation. SIAM J. Numer. Anal. 42 (2004). This scheme is formulated and analysed for the abstract Cauchy problem, where no spatial discretisation is introduced. The author shows that the numerical scheme is well-posed in H^s for sufficiently large s. But also this method seems to experience difficulties when the scheme is implemented with a a large number of spatial points.