MaGIC 2005

February 14–18, 2005

The annual informal workshop, Manifolds and Geometric Integration Colloquia (MaGIC) is organized by the Numerical Analysis groups in Bergen and Trondheim.

The workshop is intended mainly for researchers and research students in Geometric Integration, primarily from Trondheim and Bergen. The scope of the colloquia is to provide an informal atmosphere to exchange recent developments in the field of GI as well as time to establish collaborations. A restricted number of participants from abroad and often a researcher of international level is invited to give a lecture on a subject relevant to GI.

List of participants

- Håvard Berland, Norwegian University of Science and Technology, Trondheim
- Tom Bridges, University of Surrey, UK
- Elena Celledoni, Norwegian University of Science and Technology, Trondheim
- Stein Krogstad, SINTEF Applied mathematics, Norway
- Simon Malham, Heriot–Watt University, Scotland
- Borislav Minchev, Norwegian University of Science and Technology, Trondheim
- Per Christian Moan, University of Oslo, Norway
- Hans Munthe-Kaas, University of Bergen, Norway
- Jitse Niesen, Heriot–Watt University, Scotland
- Brynjulf Owren, Norwegian University of Science and Technology, Trondheim
- Niklas Sævstrøm, Norwegian University of Science and Technology, Trondheim
- Mechtild Thalhammer, University of Innsbruck, Austria
- Will Wright, Norwegian University of Science and Technology, Trondheim
- Antonella Zanna, University of Bergen, Norway

Programme

	Mon 14	**Tue 15**	**Wed 16**	**Thu 17**	**Fri 18**
07.45—08:30	Arrival	Breakfast	Breakfast	Breakfast	Breakfast
08:30—09:30		*Bridges* NEA	*Bridges* NEA	*Bridges* GI	*Bridges* MSI
09:30—10:00		Coffee	Coffee	Coffee	Coffee
10:00—11:00		*Bridges* NEA	*Bridges* NEA	*Bridges* GI	*Bridges* MSI
11:00—16:30		Free	Free	Free	Lunch/ Departure
16:30—17:15		*Thalhammer*	*Malham*	*Moan*	
17:15—17:45		Coffee	Coffee	Coffee	
17:45—18:45		*Niesen* *Wright*	*Berland* *Sævstrøm*	*Munthe-Kaas*	
20:00—21:00	Dinner	Dinner	Dinner	Dinner	
21:00—	Social activities	Social activities	Social activities	Social activities	

Abstracts

Tom Bridges

Abstract

Three distinct but related topics will be discussed. The first part, consisting of four lectures, will be on the use of exterior algebra in numerics. The particular example to be studied in detail is the numerical integration of ordinary differential equations which arise when ODEs on restricted to exterior algebra spaces. The second part, consisting of one lecture, will be on the use of geometric integration for the numerical simulation of breaking water waves. The third part, consisting of three lectures, will be on multi-symplectic numerics. Existing result on M-S numerics will be reviewed, and some new ideas for M-S numerics, based on generalizing the exterior algebra framework of part 1 to PDEs, will be discussed.

Exterior algebra and ordinary differential equations

Given a vector space V, there are a number of other vector spaces that can be built on it: the dual space V^*, the spaces of $k-$vectors $\bigwedge^k(V)$, and $k-$forms $\bigwedge^k(V^*)$, for $k = 0, \ldots, n$. Given an ordinary differential on V, it is often of interest to numerically integrate the induced systems on $\bigwedge^k(V)$ or $\bigwedge^k(V^*)$. These lectures will discuss the implementation and integration of such equations. No previous knowledge of exterior algebra will be assumed. The basics of exterior algebra will be introduced, and then applied to ODEs. A central topic is the *Grassmannian*. The Grassmannians associated with $k-$dimensional subspaces of V will be introduced and then related numerical issues discussed, including numerical preservation of the Grassmannian, and transforming so that the Grassmannian is attracting, and the role of boundary conditions. Other topics include: the various operations such as wedge and interior product, the Hodge star operator with and without a metric; extracting a basis from a $k-$vector or $k-$form; and, smoothness of subspaces.

A motivating example is the numerical integration of ODE eigenvalue problems on the real line, which arise in the study of the stability of solitary waves. Examples from this class of ODES, will be used for illustration, as well as examples from hydrodynamic stability. Another issue that arises in the study of such systems is preservation of holomorphicity in the discretization. For example, the numerical integration along a path in the complex plane, while preserving analyticity. Some aspects of discrete holomorphic functions will be discussed, such as numerical analytic continuation, and some issues of geometric integration raised.

The above theory is for general vector spaces. Two topics of interest when the vector space is symplectic is computation of the *Maslov index*, and the role of *Krein signature* in numerics. These concepts will be introduced, and then some recent results on computing the Maslov index using exterior algebra will be presented, and examples of the role of Krein signature presented.

Numerical simulation of breaking waves

The widely used governing equations for modelling water waves are Hamiltonian. The Hamiltonian structure was first identified by ZAKHAROV (1968). However this formulation is valid only for a simple free surface and is therefore not useful for breaking waves. It is not well-known that BENJAMIN & OLVER (1982) proposed a Hamiltonian formulation for any free surface, and they developed a coordinate-free formulation. In this lecture this geometric Hamiltonian formulation will be introduced and the issues associated with geometric integration discussed. The symplectic structure is non-canonical and is similar to a Lie-Poisson structure. Existing geometric integration technology does not apply, and new ideas are needed for numerical preservation of this structure. No previous knowledge of water waves will be assumed. The emphasis will be on numerical evolution of surfaces with a re-parameterization symmetry.

Multi-symplectic numerics

These lectures will begin by giving an overview of the concept of multi-symplecticity. Then previous work on *multi-symplectic integrators* will be reviewed, with particular attention to the formulations proposed by MARSDEN, PATRICK & SHKOLLER and BRIDGES & REICH, and related follow-up work by ASCHER, BAI, CHEN, GUO, LIU, MCLACHLAN, QIN, SCHOBER, WANG, YANG, ZHEN and others.

A new approach to multi-symplectic numerics will also be introduced. On the *total exterior algebra* of a Riemannian manifold there is a canonical coordinate-free multi-symplectic structure. This structure can be constructed independent of any differential equation, but is also a natural model for a class of elliptic and hyperbolic PDEs. Numerical implementation of this formulation will be presented. It is a generalization to PDEs of the numerical exterior algebra framework presented in Part 1.

Mechtild Thalhammer

Magnus type integrators for nonlinear parabolic problems

In the last years, due to the progress of the art and the increasing potentiality for the efficient calculation of the matrix exponential, the method classes of exponential and Magnus integrators received a lot of attention. In particular, owing to their favourable stability and convergence properties, numerical discretisations based on the Magnus expansion are an approved method class for the time integration of Schrödinger equations with time-dependent Hamiltonian.

In this talk, I will consider Magnus type integrators for the time discretisation of linear and nonlinear parabolic initial-boundary value problems and study their stability and convergence behaviour. For the analysis, it is convenient to interprete the partial differential equation as an abstract evolution equation on a function space employing the framework of sectorial operators and analytic semigroups on Banach spaces. The theoretical results are illustrated and confirmed by various applications and numerical examples.

Will Wright

More on GIF-methods

Krogstad introduced the generalized integrating factor (GIF) methods as a means of improving the standard integrating factor (IF) methods which suffer from bad error constants and do not preserve fixed points. In this talk we review the work of Krogstad and show how a slight modification can be used to improve the performance. We will also discuss the stiff order conditions and show the stiff order for the GIF and the modified GIF methods. Some numerical experiments will be included.

Jitse Niesen

On the stability of the Magnus method

We briefly review the classical stability theory for Runge–Kutta methods and comment on its applicability to the Magnus method. Then, we consider the behaviour of the fourth-order Magnus method on a particular linear and mildly nonautonomous equation. The matrix in the equation has one eigenvalue around zero and one very negative eigenvalue. We calculate estimates for the local and global error and conclude that order reduction takes place. The analysis is supported by numerical experiments.

Simon Malham

Efficient high order stochastic integrators for linear system

We present efficient high order numerical schemes for the strong solution of linear stochastic differential equations. These schemes are based on the Neumann and Magnus expansions and the efficient evaluation of high order multiple Stratonovich integrals. Comparing these methods to a classical stochastic numerical integration scheme, we demonstrate several orders of magnitude improvement in accuracy for the same computational cost. To indicate their preferential use in some dynamic programming applications, we apply them to a stochastic Riccati differential system that can be linearized.

Håvard Berland

Exponential Lawson for the non-linear Schrödinger equation

We explore the numerical properties of the the Exponential fourth order Lawson integrator on the non-linear Schrödinger equation for varying regularity of the potential and the initial condition. Some estimates on the regularity dependency are presented.

Niklas Sævstrøm

Solutions for the Euler equations

It has been know since the time of Jacobi that the solution to the free rigid body (FRB) equations of motions is given in terms of a certain type of elliptic functions. It is an interesting question to ask whether these functions can be calculated in such a way that they yield a faster and more accurate solutions to the FRB equations than standard numerical ODE solvers.

In this talk we give a brief review of the derivation of the solution to the FRB equations in terms of the Jacobi elliptic functions. In particular we will recount how these function can be efficiently approximated, and show some numerical tests which compares the performance with the method of McLachlan and Reich. Finally, we will discuss possible extensions and limitations of this approach.

Per Christian Moan

Some recent results on time-averaging/Magnus-series

Hans Munthe-Kaas

Diffusion tensor imaging, matrix valued images and geometric integrators