Marked Point Models with Complex Conditioning Used for Modelling of Shales

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Abstract

Shales with low permeability will create barriers for fluid flow in a sandstone reservoir. The location of shales has great importance on the flow properties of the reservoir. A two dimensional stochastic model for the shale distribution in a sandstone reservoir is established.

An extension of a marked point process is used to model shales in order to satisfy conditions from observations in wells. A Gaussian field is placed at the top and bottom of the shales. In this way, it is possible to find a realisation that fits the observations from more than one well exactly. In an ordinary marked point model, with constant height of the shales, this would be impossible.

A model formulation for a reservoir with several shales is established. A simulation algorithm is described, and a simple numerical example is presented.

1 Introduction

An oil reservoir may consist of sandstone and shales. Shales have low permeability, and will create barriers for fluid flow, while sandstone has high permeability, and allows fluids to flow easily. In order to characterize the flow properties of the reservoir, it is therefore of great interest to locate the shales. Shales can appear in different sizes and shapes. The geometry and size of the shales, and the sand shale ratio will have influence on the flow properties in the reservoir.

Information about the reservoir characteristics is obtained from observations in the wells and from geological knowledge and experience. The sand shale ratio and the depths on which the shales are located are indicated by the wells. Geological experience can tell whether shales observed in different wells are parts of the same shale, or not.

Based on this information, a stochastic model for the reservoir will be created. A good model should be representative for the reservoir under study. Information about the reservoir should be used in a suitable way, and the model should be able to reproduce observations in the wells. Central references on this field are Haldorsen and Lake (1984), Haldorsen and Damsleth (1990) and Omre (1992).

The shale units in a reservoir can be modelled by using an ordinary marked point model in which each shale is represented by a marked point of two dimensions. See Stoyan et.al. (1987). The sand will constitute the background.

Let the marked point in two dimensions be denoted

$$\boldsymbol{U} = (\boldsymbol{C}, \boldsymbol{S}),$$

with $\boldsymbol{C} = (C^1, C^2)$ being the location of the point, and $\boldsymbol{S}$ being a set of marks, for example $\boldsymbol{S} = (L, H)$, constant length and height of the shale. The stochastic model for n shales over a domain $\mathcal{D}$ is defined by the joint probability distribution

$$f_{\boldsymbol{U}_1,\ldots,\boldsymbol{U}_n}(\boldsymbol{u_1}, \ldots, \boldsymbol{u_n}).$$

The height of the shales being intersected can be observed in the wells. If a shale is intersected by more than one well, the observed height may differ from one well to the other. The marked point model defined above assigns a constant height to each shale. With this model, it will not be possible to fit the observations from more than one well, and hence

$$f_{\boldsymbol{U}_1,\ldots,\boldsymbol{U}_n}(\boldsymbol{u_1}, \ldots, \boldsymbol{u_n} | \boldsymbol{o}) = \boldsymbol{0} \quad \text{for all} \quad (\boldsymbol{u_1}, \ldots, \boldsymbol{u_n})$$

with $\boldsymbol{o}$ being the observation vector. This means that the model will not have flexibility to reproduce observed data from the wells.

The objective of this article is to find a model that is able to handle this conditioning problem. The marked point model will be extended to contain a residual random field on the top and bottom surface of the shales. Hence the height of the shales will no longer be constant.

2 Model definition

The model presented is a two-dimensional model, but the idea can easily be extended to higher dimensions. The horizontal direction is denoted x_1 and the vertical direction is denoted x_2, see figure 1. Each shale will now be modelled as

$$\boldsymbol{U} = (\boldsymbol{C}, \boldsymbol{Z}(x_1), \boldsymbol{Y}(x_2), \boldsymbol{S}) \tag{1}$$

with

- $\boldsymbol{C} = (C^1, C^2)$ denoting position of the centerpoint,

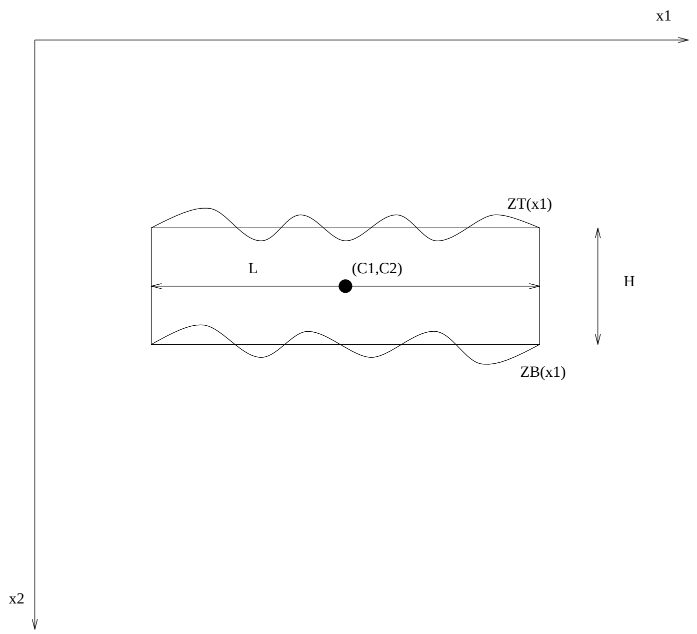

Figure 1: *Model for a single shale.*

- $\boldsymbol{Z}(x_1) = (\boldsymbol{Z}_B(x_1), \boldsymbol{Z}_T(x_1))$ being the top and bottom position of the shale,
- $\boldsymbol{Y}(x_2)$ being the length of the shale, and
- $\boldsymbol{S} = (L, H)$ being the expected length and height of the shale.

The length of the shale $\boldsymbol{Y}(x_2)$ is assumed to be symmetric around C^1, and constant equal to L for all possible values of x_2. In order to simplify notation, let the parameter vector be denoted $\boldsymbol{\Theta} = (\boldsymbol{C}, \boldsymbol{S}) = (C^1, C^2, L, H)$. One can now write $\boldsymbol{U} = (\boldsymbol{\Theta}, \boldsymbol{Z}(x_1), \boldsymbol{Y}(x_2))$.

The distribution of $\boldsymbol{U}$ will be developed in a Bayesian setting.

2.1 Prior Model

Consider the set of n shales within a domain $\mathcal{D}$. The parameter vector $\underline{\boldsymbol{\Theta}} = (\boldsymbol{\Theta_1}, \ldots, \boldsymbol{\Theta_n})$ can be regarded as an ordinary marked point model with prior probability density $f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}})$ which is specified based on prior geological experience. One possible expression for $f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}})$ is,

$$f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}}) = const \times \exp\{\sum b(\boldsymbol{\theta_i}) + \sum\sum d(\boldsymbol{\theta_i}, \boldsymbol{\theta_j})\}$$

entailing only pairwise interaction between shales. The probability density for an arbitrary shale i can be expressed by,

$$f_{\boldsymbol{\Theta_i}}(\boldsymbol{\theta_i}) = \int_{\Omega_{-i}} f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}}) d\boldsymbol{\theta}_{-i}$$

where $d\boldsymbol{\theta}_{-i} = d\boldsymbol{\theta}_{\mathbf{1}} \ldots, d\boldsymbol{\theta}_{i-1}, d\boldsymbol{\theta}_{i+1}, \ldots, d\boldsymbol{\theta}_{\boldsymbol{n}}$. The associated random variable $\boldsymbol{Z}_{\boldsymbol{i}}(x_1) = (\boldsymbol{Z}_{iB}(x_1), \boldsymbol{Z}_{iT}(x_1))$ is assumed to be a Gaussian field with the following distribution conditioned on the parameter vector $\boldsymbol{\Theta}_{\boldsymbol{i}}$

$$\boldsymbol{Z}_{\boldsymbol{i}}(x_1)|\boldsymbol{\Theta}_{\boldsymbol{i}} = \boldsymbol{\theta}_{\boldsymbol{i}} \sim N\left(\begin{array}{c} c_i^2 + h_i/2 \\ c_i^2 - h_i/2 \end{array}, \left(\begin{array}{cc} \sigma^2 & \tau \\ \tau & \sigma^2 \end{array} \right) \rho(x_1' - x_1'') \right). \tag{2}$$

for $x_1 \in [c_i^1 \pm l_i/2]$ and undefined otherwise.

Note that the expected top and bottom is centered around C^2, the variance is identical to σ^2 with spatial correlation function $\rho(\cdot)$. The correlation between top and bottom is τ.

Combining $f_{\boldsymbol{\Theta}_{\boldsymbol{i}}}(\boldsymbol{\theta}_{\boldsymbol{i}})$ and $f_{\boldsymbol{Z}_{\boldsymbol{i}}|\boldsymbol{\Theta}_{\boldsymbol{i}}}(\boldsymbol{z}(x_1)|\boldsymbol{\theta}_{\boldsymbol{i}})$ provides the distribution for an arbitrary shale i:

$$f_{\boldsymbol{Z}_{\boldsymbol{i}}}(\boldsymbol{z}_{\boldsymbol{i}}(x_1)) = \int_{\Omega} f_{\boldsymbol{Z}_{\boldsymbol{i}}|\boldsymbol{\Theta}_{\boldsymbol{i}}}(\boldsymbol{z}_{\boldsymbol{i}}(x_1)|\boldsymbol{\theta}_{\boldsymbol{i}}) f_{\boldsymbol{\Theta}_{\boldsymbol{i}}}(\boldsymbol{\theta}_{\boldsymbol{i}}) d\boldsymbol{\theta}_{\boldsymbol{i}}. \tag{3}$$

Assuming that the stochastic fields $\boldsymbol{Z}_{\boldsymbol{i}}(x_1)|\boldsymbol{\Theta}_{\boldsymbol{i}}$ in different shales are conditionally independent, the following probability density for a reservoir containing n shales represented by $\underline{\boldsymbol{Z}}(x_1) = (\boldsymbol{Z}_{\mathbf{1}}(x_1), \ldots, \boldsymbol{Z}_{\boldsymbol{n}}(x_1))$, is obtained:

$$f_{\underline{\boldsymbol{Z}}}(\underline{\boldsymbol{z}}(x_1)) = \int_{\Omega} f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}}) \prod_{i=1}^{n} f_{\boldsymbol{Z}_{\boldsymbol{i}}|\boldsymbol{\Theta}_{\boldsymbol{i}}}(\boldsymbol{z}_{\boldsymbol{i}}(x_1)|\boldsymbol{\theta}_{\boldsymbol{i}}) d\boldsymbol{\theta}. \tag{4}$$

Recall that the length of shales, $\boldsymbol{Y}(x_2)$, is defined to be identical to the parameter L. Hence the model for $\boldsymbol{Y}(x_2)$, $f_{\underline{\boldsymbol{Y}}}(\underline{\boldsymbol{y}}(x_2))$ requires no additional parameters.

2.2 Observations

Observations are collected from the wells in the reservoir. It is assumed that all wells are vertical, and in total there are N wells at the locations $x_{11}, \ldots, x_{1N}$.

The depth observations are contained in the vector $\boldsymbol{o}$, and the observations associated with shale number i are presented in figure 2 and defined by:

- Index of the m_i wells which penetrate the shale, and the location of these wells are denoted $w_{1i}, \ldots, w_{m_i i}$.
- The observed depths to top and bottom of the shales in the m_i penetrating wells, $z_{iT}(w_{i1}), \ldots, z_{iT}(w_{m_i i})$ and $z_{iB}(w_{i1}), \ldots, z_{iB}(w_{m_i i})$.

In addition, the location of the non-penetrating wells will give information about the length of the shale. In the case of a shale not penetrated by any well, this is in fact the only information available.

It is assumed that no overlap between shales is observed in the wells. Moreover it is assumed that there is no conflicting information. For example, if two wells are intepreted to penetrate the same shale, then all wells between them must penetrate the same shale.

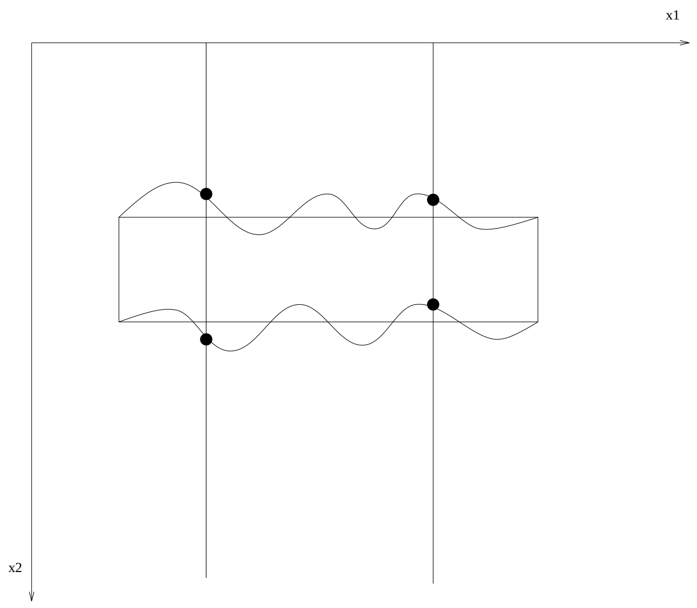

Figure 2: *Observations for shale number i with $m_i = 2$.*

2.3 Posterior Model

Given the observations, the conditional probability density for top and bottom of shale number i, $\boldsymbol{Z_i}(x_1)$, is as follows.

$$f_{\boldsymbol{Z_i}|\boldsymbol{O}}(\boldsymbol{z_i}(x_1)|\boldsymbol{o}) = \int_{\Omega} f_{\boldsymbol{Z_i}|\boldsymbol{\Theta_i},\boldsymbol{O}}(\boldsymbol{z_i}(x_i)|\boldsymbol{\theta_i},\boldsymbol{o}) f_{\boldsymbol{\Theta_i}|\boldsymbol{O}}(\boldsymbol{\theta_i}|\boldsymbol{o}) d\boldsymbol{\theta_i} \tag{5}$$

Analogous to expression (4), the model for n shales is

$$f_{\underline{\boldsymbol{Z}}|\boldsymbol{O}}(\underline{\boldsymbol{z}}(x_1)|\boldsymbol{o}) = \int_{\Omega} f_{\underline{\boldsymbol{\Theta}}|\boldsymbol{O}}(\underline{\boldsymbol{\theta}}|\boldsymbol{o}) \prod_{i=1}^{n} f_{\boldsymbol{Z_i}|\boldsymbol{\Theta_i},\boldsymbol{O}}(\boldsymbol{z_i}(x_1)|\boldsymbol{\theta_i},\boldsymbol{o}) d\boldsymbol{\theta} \tag{6}$$

Recall that the parameter vector is $\boldsymbol{\Theta} = (C^1, C^2, L, H)$. The posterior distribution for the horizontal (C^1, L) and vertical (C^2, H) parameters are determined independently.

The vertical parameters C^2 and H can be conditioned directly on the observations in the wells. It is possible to find analytical expressions for the posterior distribution. The calculations for a special case considering independent C^2 and H is found in Appendix A. Assuming Gaussian prior, the posterior will also be Gaussian. The mean and variance for H in shale number i is

$$E(H_i|\boldsymbol{Z}_{di}) = \mu_{H_i} + \sigma^2_{H_i}\boldsymbol{F}^T(\boldsymbol{C} + \sigma^2_{H_i}\boldsymbol{F}\boldsymbol{F}^T)^{-1}(\boldsymbol{Z}_{di} - \boldsymbol{F}\mu_{H_i}) \tag{7}$$

$$Var(H_i|\boldsymbol{Z}_{di}) = \sigma^2_{H_i} - \sigma^2_{H_i}\boldsymbol{F}^T(\boldsymbol{C} + \sigma^2_{H_i}\boldsymbol{F}\boldsymbol{F}^T)^{-1}\sigma^2_{H_i}\boldsymbol{F} \tag{8}$$

with μ_{H_i} being the prior mean, $\sigma^2_{H_i}$ being the prior variance, $F = [1, \ldots, 1]^T$ being a vector of dimension m_i, $\boldsymbol{C}_{kj} = 2\rho(w_{ki} - w_{ji})(\sigma^2 - \tau)$, and

$$\boldsymbol{Z}_{di} = \left[\begin{array}{l} z_{iB}(w_{1i}) - z_{iT}(w_{1i}) \\ \vdots \\ z_{iB}(w_{m_i i}) - z_{iT}(w_{m_i i}) \end{array} \right].$$

The mean and variance for C^2 is

$$E(C_i^2|\boldsymbol{Z}_{si}) = \mu_{C_i^2} + \sigma^2_{C_i^2}\boldsymbol{F}^T(\boldsymbol{C} + \sigma^2_{C_i^2}\boldsymbol{F}\boldsymbol{F}^T)^{-1}(\boldsymbol{Z}_{si} - \boldsymbol{F}\mu_{C_i^2}) \tag{9}$$

$$Var(C_i^2|\boldsymbol{Z}_{si}) = \sigma^2_{C_i^2} - \sigma^2_{C_i^2}\boldsymbol{F}^T(\boldsymbol{C} + \sigma^2_{C_i^2}\boldsymbol{F}\boldsymbol{F}^T)^{-1}\sigma^2_{C_i^2}\boldsymbol{F} \tag{10}$$

with $\mu_{C_i^2}$ being the prior mean, $\sigma_{C_i^2}$ denoting the prior variance, $F = [2, \ldots, 2]$, $\boldsymbol{C}$ being the same as for H, and

$$\boldsymbol{Z}_{si} = \left[\begin{array}{l} z_{iB}(w_{1i}) + z_{iT}(w_{1i}) \\ \vdots \\ z_{iB}(w_{m_i i}) + z_{iT}(w_{m_i i}) \end{array} \right].$$

For shales not penetrated by any well, the posterior distribution is identical to the prior distribution.

For the horizontal parameters C^1 and L the situation is more complicated. In Chessa (1992) it was pointed out that the wells oversamples large shales, but the multi-well problem was not discussed. The wells give information about maximum and minimum length of shales penetrated by two or more wells, and maximum length otherwise, given the center location C^1. No observation of the exact length is available. It is complicated to find the posterior distribution $f_{L,C^1|\boldsymbol{O}}(l, c^1|\boldsymbol{o})$ analytically, but realisations can be generated by using rejection sampling, see Ripley (1987).

The simplest way of doing this is to sample from the prior distribution $f_{L,C^1}(l, c^1)$ and accept the realisation with probability

$$\begin{aligned} p &\propto \frac{f_{L,C^1|\boldsymbol{O}}(l, c^1|\boldsymbol{o})}{f_{L,C^1}(l, c^1)} = \frac{f_{\boldsymbol{O}|L,C^1}(\boldsymbol{o}|l, c^1) f_{L,C^1}(l, c^1)}{f_{L,C^1}(l, c^1) f_{\boldsymbol{O}}(\boldsymbol{o})} \\ &\propto f_{\boldsymbol{O}|L,C^1}(\boldsymbol{o}|l, c^1) = \begin{cases} 1 & \text{conditioning is fulfilled} \\ 0 & \text{conditioning is not fulfilled} \end{cases} \end{aligned}$$

This procedure can result in a very low acceptance rate. In order to increase this acceptance rate, an importance sampling approach should be used. Sample from the distribution $f^o_{L,C^1|\boldsymbol{O}}(l, c^1|\boldsymbol{o}) = f^o_{C^1|\boldsymbol{O}}(c^1|\boldsymbol{o}) f_{L|C^1}(l|c^1)$ with $f^o_{C^1|\boldsymbol{O}}(c^1|\boldsymbol{o})$ being uniformly distributed over the region for which $f_{C^1|\boldsymbol{O}}(c^1|\boldsymbol{o})$ is non-zero. The acceptance probability will then be:

$$\begin{aligned} p &\propto \frac{f_{L,C^1|\boldsymbol{O}}(l, c^1|\boldsymbol{o})}{f^o_{L,C^1|\boldsymbol{O}}(l, c^1|\boldsymbol{o})} = \frac{f_{\boldsymbol{O}|L,C^1}(\boldsymbol{o}|l, c^1) f_{C^1}(c^1) f_{L|C^1}(l|c^1)}{f^o_{C^1|\boldsymbol{O}}(c^1|\boldsymbol{o}) f_{L|C^1}(l|c^1) f_{\boldsymbol{O}}(\boldsymbol{o})} \\ &\propto f_{\boldsymbol{O}|L,C^1}(\boldsymbol{o}|l, c^1) f_{C^1}(c^1) \propto \begin{cases} f_{C^1}(c^1) & \text{conditioning is fulfilled} \\ 0 & \text{conditioning is not fulfilled} \end{cases} \end{aligned}$$

In practice, the simulation for a single shale is done as follows:

1. Find the interval where c^1 has a positive probability of being accepted.
2. Sample c^1 from a uniform distribution on this interval.
3. Sample a length l from the prior distribution of L.
4. Accept the realisation with probability $f_{C^1}(c^1)$ if the conditioning is fulfilled, otherwise go to 2.

3 Example

Consider a reservoir with n shales and independent model parameters. Then the prior distribution for the parameter vector $\boldsymbol{\Theta_i}$ for shale number i, see section 2.1, is given by

$$f_{\boldsymbol{\Theta_i}}(\boldsymbol{\theta_i}) = f_{C_i^1}(c_i^1) f_{C_i^2}(c_i^2) f_{L_i}(l_i) f_{H_i}(h_i).$$

For a reservoir with n shales, the joint probability density for the parameters $\underline{\boldsymbol{\Theta}} = (\boldsymbol{\Theta_1}, \ldots, \boldsymbol{\Theta_n})$ is given by

$$f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}}) = const \times \exp\{\sum b(\boldsymbol{\theta_i}) + \sum\sum d(\boldsymbol{\theta_i}, \boldsymbol{\theta_j})\}$$

with $d(\boldsymbol{\theta_i}, \boldsymbol{\theta_j})$ representing the interaction between shales.
Assuming independence between parameters belonging to the same shale, gives

$$b^{\boldsymbol{\Theta_i}}(\boldsymbol{\theta_i}) = b^{C_i^1}(c_i^1) + b^{C_i^2}(c_i^2) + b^{L_i}(l_i) + b^{H_i}(h_i).$$

It is natural to assume that interaction between shales only involve the location parameters (C^1, C^2), that is length and height of a shale is not influencing the position of the other shales. This leads to

$$f_{\underline{\boldsymbol{\Theta}}}(\underline{\boldsymbol{\theta}}) = const \times \exp\{\sum [b^{C_i^1}(c_i^1) + b^{C_i^2}(c_i^2) + b^{L_i}(l_i) + b^{H_i}(h_i)] + \sum_{l=1}^{2}\sum_{j=1}^{n}\sum_{i=1}^{n} d(c_i^l, c_j^l)\}.$$

The interaction function $d(\cdot,\cdot)$ is chosen in a way such that neighbouring shales tend to repulse each other. In the simulation example described later, the following function is used.

$$d(x_i, x_j) = \begin{cases} -\infty & \text{if } |x_i - x_j| \le r_{min} \\ \ln(\frac{|x_i - x_j| - r_{min}}{r_{max} - r_{min}}) & \text{if } r_{min} < |x_i - x_j| < r_{max} \\ 0 & \text{if } |x_i - x_j| > r_{max} \end{cases} \tag{11}$$

Realisations from the marginal distributions for the parameters can be found as shown in the previous section. But the simultaneous conditional distribution $f_{\underline{\boldsymbol{\Theta}}|\boldsymbol{O}}(\underline{\boldsymbol{\theta}}|\boldsymbol{o})$ can not be determined analytically, and realisations must be generated by simulation.

3.1 Simulation Algorithm

The simulation in the example is done in two steps, and the procedure is as follows:

1. Find a realisation of $\boldsymbol{\Theta}|\boldsymbol{O}$ by using a continous version of the Metropolis-Hastings algorithm.
2. Using the realisation of $\boldsymbol{\Theta}|\boldsymbol{O}$ from step 1, find a realisation of $f_{\boldsymbol{Z_i}|\boldsymbol{\Theta}}(\boldsymbol{Z_i}(x_1))|\boldsymbol{\theta},\boldsymbol{o})$ for each shale, by using geostatistical simulation, and a realisation of $\boldsymbol{Y_i}(x_2)|\boldsymbol{O}$ directly.

Step 1

A continous version of the Metropolis-Hastings algorithm is used, see Metropolis et. al. (1953), Hastings (1970) and Geyer and Møller (1993). In order to get fast convergence of the Metropolis-Hastings algorithm, it is important to choose the transition probability matrix $\boldsymbol{Q}$ in a suitable way. The transition probability to states that are impossible to reach due to the conditioning should be zero. Each transition will alter only one shale.

If the following states are defined

$$i = (\boldsymbol{\theta_1}|\boldsymbol{o},\ldots,\boldsymbol{\theta_k}|\boldsymbol{o},\ldots,\boldsymbol{\theta_n}|\boldsymbol{o})$$

and

$$j = (\boldsymbol{\theta_1}|\boldsymbol{o},\ldots,\boldsymbol{\theta_k}'|\boldsymbol{o},\ldots,\boldsymbol{\theta_n}|\boldsymbol{o})$$

the transition probability is set to

$$\begin{aligned} q_{ij} &= f_{\boldsymbol{\Theta}|\boldsymbol{O}}(\boldsymbol{\theta}'|\boldsymbol{o}) = f_{H|\boldsymbol{O}}(h_k'|\boldsymbol{o}) f_{C^2|\boldsymbol{O}}(c_k^{2'}) f_{C^1,L|\boldsymbol{O}}(c_k^{1'},l_k'|\boldsymbol{o}) \\ &= const \times \exp(b^{C_1,L}(c_k^{1'},l_k'|\boldsymbol{o}) + b^{C_2}(c_k^{2'}|\boldsymbol{o}) + b^H(h_k'|\boldsymbol{o})). \end{aligned} \tag{12}$$

This is the marginal posterior distribution for a single shale. The vector $\boldsymbol{c_k} = [c_k^1, c_k^2]$ will now denote the centerpoint for shale number k.

According to the algorithm, this gives the acceptance probability

$$\alpha_{ij} = s_{ij}[1 + \frac{f_{\boldsymbol{\Theta}|\boldsymbol{O}}(i|\boldsymbol{o})\exp b(\boldsymbol{\theta_k}'|\boldsymbol{o})}{f_{\boldsymbol{\Theta}|\boldsymbol{O}}(j|\boldsymbol{o})\exp b(\boldsymbol{\theta_k}|\boldsymbol{o})}]^{-1} = \frac{s_{ij}}{1+\exp(\sum_i d(\boldsymbol{c_k}',\boldsymbol{c_i}) - d(\boldsymbol{c_k},\boldsymbol{c_i}))} \tag{13}$$

If s_{ij} is chosen as large as possible

$$s_{ij} = min\{1 + \frac{\exp(\sum_i d(\boldsymbol{c_k},\boldsymbol{c_i}))}{\exp(\sum_i d(\boldsymbol{c_k}',\boldsymbol{c_i}))}, 1 + \frac{\exp(\sum_i d(\boldsymbol{c_k}',\boldsymbol{c_i}))}{\exp(\sum_i d(\boldsymbol{c_k},\boldsymbol{c_i}))}\},$$

the acceptance probability becomes

$$\alpha_{ij} = min\{1, \frac{1+\exp(-\sum_i d(\boldsymbol{c_k},\boldsymbol{c_i}) + \sum_i d(\boldsymbol{c_k}',\boldsymbol{c_i}))}{1+\exp(-\sum_i d(\boldsymbol{c_k}',\boldsymbol{c_i}) + \sum_i d(\boldsymbol{c_k},\boldsymbol{c_i}))}\} \tag{14}$$

Note that α_{ij} only depends on the interaction between the shales, while the transition probability only depends on the conditional marginal distribution of the shales.

The algorithm is as follows: Initiation is made by constructing n shales independently by drawing from the conditional marginal distribution for each shale. Iterate a preassigned number of times:

1. Select an arbitrary shale k.
2. Generate new parameters for shale k, and choose $\boldsymbol{\theta_k}'$ with probability q_{ij} given by expression (12).
3. Replace $\boldsymbol{\theta_k}$ by $\boldsymbol{\theta_k}'$ with probability α_{ij} given by expression (14).

Step 2

Given a set of model parameters, $\underline{\boldsymbol{\theta}}$, realisations of $\boldsymbol{Z_i}(x_1)|\boldsymbol{\theta}, \boldsymbol{o}$, $i = 1, \ldots, n$, can be found by conditional simulation as defined in geostatistics, see Journel and Huijbregts (1978). This gives a realisation of the conditional model, which ensures that the observations are reproduced.

For shales not penetrated by wells, the fields will of course be simulated from the prior distribution given by expression (2).

Given a set of model parameters, $\underline{\boldsymbol{\theta}}$, realisations of $(\boldsymbol{Y_i}(x_2)|\boldsymbol{\theta}, \boldsymbol{o})$, $i = 1, \ldots, n$ can be obtained by setting $y_i(x_2) = l_i$, $i = 1, \ldots, n$.

3.2 Numerical Results

A simple simulation example with 10 shales is done. Five shales are penetrated by different numbers of wells, while the other five are not penetrated. The centerpoint in horizontal direction has a uniform prior distribution, while the other parameters have Gaussian prior distribution with the following parameters:

	μ	σ
C_2	10.0	5.0
H	1.0	0.5
L	18.0	8.0

In expression (2), σ is set to 0.8 and τ to 0.2, and the correlation function is defined to be $\rho(x) = e^{-0.5|x|}$. In the interaction function in expression (11), r_{max} is set to 30.0 and r_{min} to 2.0. The length of the reservoir is set to 100. Two realisations of this reservoir is shown in figure 3.

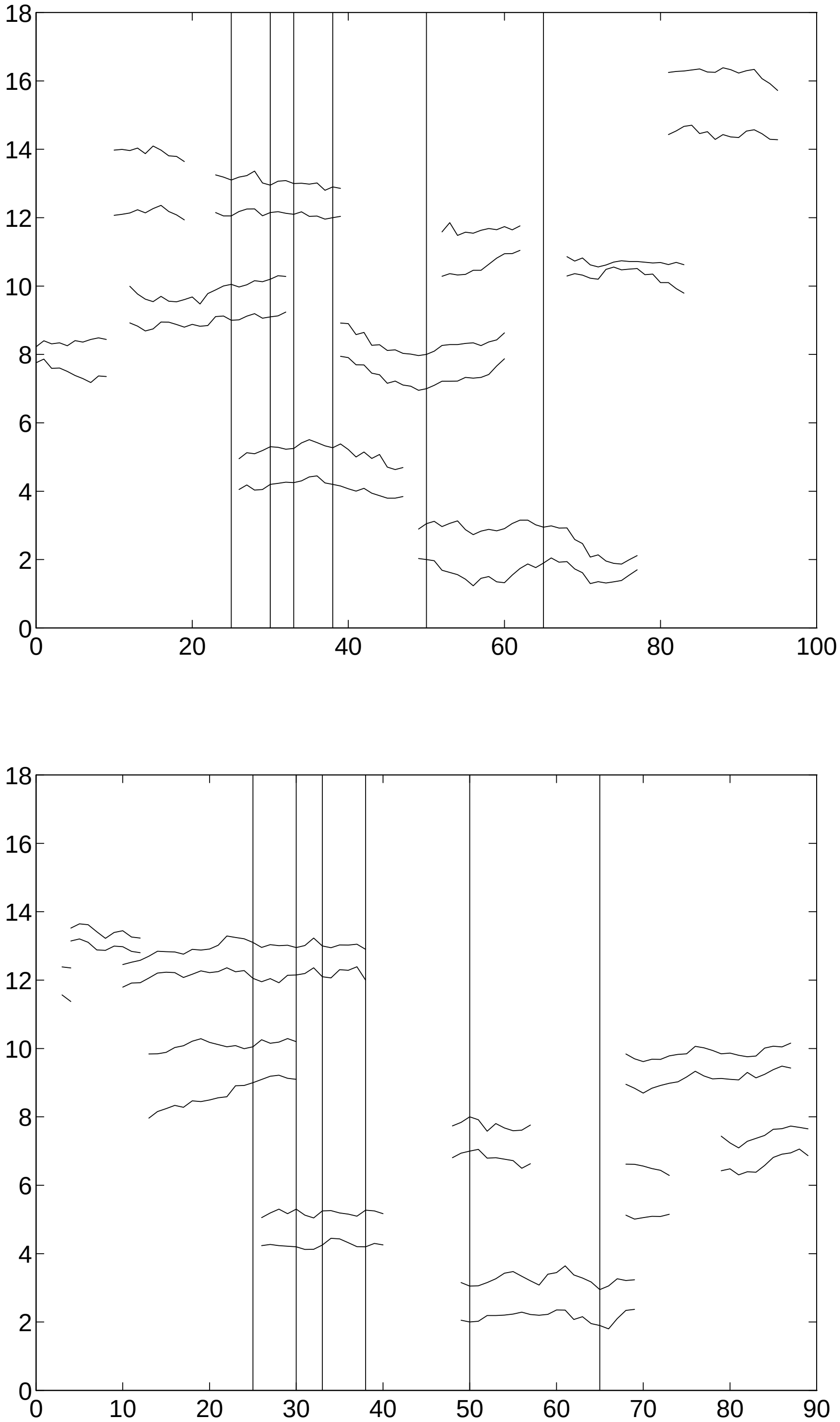

Figure 3: *Simulated reservoir conditioned on observations in wells.*

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A Posterior Distribution for the Parameters

Gaussian random functions in a Bayesian setting is presented in Omre and Halvorsen (1989), and the developments that follows is a special case of their model.

Let B_l, $l = 1, \ldots, L$ be Gaussian parameters with expectation $E(B_l) = \mu_l$ and covariance $Cov(B_l, B_k) = \sigma_{lk}$. Define a Gaussian random function

$$Z(x) = \sum_{l=1}^{L} B_l f^l(x) + R(x)$$

and let

$$E(Z(x)|B_l; l = 1, \ldots, L) = \sum_{l=1}^{L} B_l f^l(x)$$

$$Cov(Z(x'), Z(x'')|B_l; l = 1, \ldots, L) = C_{Z|B}(x' - x'')$$

The inetention is to find the posterior distribution of the parameters, given the observations $Z(x_i)$; $i = 1, \ldots, N$.

Linear estimators is considered:

$$E(B_l|Z(x_i); i = 1, \ldots, N) = \sum_{i=1}^{N} \beta_i^l Z^T(x_i) + \mu_l$$

where N is the number of observations, β_i^l are weights to be determined, and $Z^T(x_i) = Z(x_i) - \sum \mu_l f^l(x_i)$; $i = 1, \ldots, N$ are the centered observations.

The weights $\boldsymbol{\beta}^l$ are found by minimizing the variance, $Var[B_l - E(B_l|\boldsymbol{Z})]$ with respect to $\boldsymbol{\beta}^l$. The following notation is now introduced: An $N \times L$ matrix $\boldsymbol{\beta}$ with elements $\boldsymbol{\beta}_{li} = \beta_l^i$, a vector $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_L]$, an $N \times L$ matrix $\boldsymbol{F}$ with elements $\boldsymbol{F}_{li} = f^l(x_i)$. The vector $\boldsymbol{Z}$ contains the observations $Z(x_1), \ldots, Z(x_N)$. The $N \times N$ matrix $\boldsymbol{C}$ has elements $\boldsymbol{C}_{ij} = C_{Z|B}(x_i - x_j)$, and the $L \times L$ matrix $\boldsymbol{\Sigma}$ has elements $\boldsymbol{\Sigma}_{lk} = \sigma_{lk}$.

Written in matrix form, the solution fulfills the following equation:

$$(\boldsymbol{C} + \boldsymbol{F}\boldsymbol{\Sigma}\boldsymbol{F}^T)\boldsymbol{\beta} = \boldsymbol{F}\boldsymbol{\Sigma}$$

This gives $\boldsymbol{\beta} = (\boldsymbol{C} + \boldsymbol{F}\boldsymbol{\Sigma}\boldsymbol{F}^T)^{-1}\boldsymbol{F}\boldsymbol{\Sigma}$, and writing $\boldsymbol{B} = [B_1, \ldots, B_L]$ one gets

$$E(\boldsymbol{B}|\boldsymbol{Z}) = \boldsymbol{\mu} + \boldsymbol{\beta}^T(\boldsymbol{Z} - \boldsymbol{F}\boldsymbol{\mu}) = \boldsymbol{\mu} + (\boldsymbol{F}\boldsymbol{\Sigma})^T(\boldsymbol{C} + \boldsymbol{F}\boldsymbol{\Sigma}\boldsymbol{F}^T)^{-1}(\boldsymbol{Z} - \boldsymbol{F}\boldsymbol{\mu}) \tag{15}$$

$$\begin{aligned} Cov(\boldsymbol{B}|\boldsymbol{Z}) &= E(Cov(\boldsymbol{B}|\boldsymbol{Z})) = Cov(\boldsymbol{B}) - Cov(E(\boldsymbol{B}|\boldsymbol{Z})) \\ &= \boldsymbol{\Sigma} - (\boldsymbol{F}\boldsymbol{\Sigma})^T(\boldsymbol{C} + \boldsymbol{F}\boldsymbol{\Sigma}\boldsymbol{F}^T)^{-1}Cov(\boldsymbol{Z})(\boldsymbol{C} + \boldsymbol{F}\boldsymbol{\Sigma}\boldsymbol{F}^T)^{-1}(\boldsymbol{F}\boldsymbol{\Sigma}) \\ &= \boldsymbol{\Sigma} - (\boldsymbol{F}\boldsymbol{\Sigma})^T(\boldsymbol{C} + \boldsymbol{F}\boldsymbol{\Sigma}\boldsymbol{F}^T)^{-1}(\boldsymbol{F}\boldsymbol{\Sigma}) \end{aligned} \tag{16}$$

If the prior distribution is Gaussian, then the posterior is Gaussian too, and is therefore completely determined.

A noninformative prior entails that the eigenvalues of the covariance matrix $\boldsymbol{\Sigma}$ goes to infinity. Then it can be shown that $E(\boldsymbol{B}|\boldsymbol{Z}) \to (\boldsymbol{F}^T\boldsymbol{C}^{-1}\boldsymbol{F})^{-1}\boldsymbol{F}^T\boldsymbol{C}^{-1}\boldsymbol{Z}$ and $Cov(\boldsymbol{B}|\boldsymbol{Z}) \to (\boldsymbol{F}^T\boldsymbol{C}^{-1}\boldsymbol{F})^{-1}$.

Suitable functions $Z(x)$ must be found in order to solve the problem at hand. Using $Z_T(x)$ and $Z_B(x)$ separately gives different results. Combinations of these two random functions will be used. The posterior distribution for H_i and C_i^2 are found separately using different $Z(x)$ functions. This can be done because it is assumed that H_i and C_i^2 are independent of each other.

Posterior distribution for H_i

As $Z(x)$-function, use

$$Z_{di}(x)|\boldsymbol{\Theta} = Z_{iB}(x)|\boldsymbol{\Theta} - Z_{iT}(x)|\boldsymbol{\Theta} \sim N(H_i, 2\rho(x_1' - x_1'')(\sigma^2 - K)).$$

Let $B_1 = H_i$, $f^1(x) = 1$, $B_1 = f^i(x) = 0$, $i = 2, 3, \ldots$. Assume that shale number i is penetrated by m_i wells. The observation vector

$$\boldsymbol{Z}_{di} = \begin{bmatrix} z_{iB}(w_{1i}) - z_{iT}(w_{1i}) \\ \vdots \\ z_{iB}(w_{m_i i}) - z_{iT}(w_{m_i i}) \end{bmatrix}$$

and the vector $F = [1, \ldots, 1]^T$ are both of length m_i. The matrix $\boldsymbol{\Sigma}$ will now be a scalar σ_{Hi}^2 because only one parameter is present, and $\boldsymbol{C}_{kj} = 2\rho(w_{ki} - w_{ji})(\sigma^2 - \tau)$.

Using expression (15) one gets

$$E(H_i|\boldsymbol{Z}_{di}) = \mu_{H_i} + \sigma_{H_i}^2\boldsymbol{F}^T(\boldsymbol{C} + \sigma_{H_i}^2\boldsymbol{F}\boldsymbol{F}^T)^{-1}(\boldsymbol{Z}_{di} - \boldsymbol{F}\mu_{H_i}) \tag{17}$$

Using (16) gives

$$Var(H_i|\boldsymbol{Z}_{di}) = \sigma_{H_i}^2 - \sigma_{H_i}^2\boldsymbol{F}^T(\boldsymbol{C} + \sigma_{H_i}^2\boldsymbol{F}\boldsymbol{F}^T)^{-1}\sigma_{H_i}^2\boldsymbol{F} \tag{18}$$

If a noninformative prior is used, $E(H_i|\boldsymbol{Z}_{di}) = (\boldsymbol{F}^T\boldsymbol{F})^{-1}\boldsymbol{F}^T\boldsymbol{Z}_{di}$, and $Var(H_i|\boldsymbol{Z}_{di}) = \sigma_{H_i}^2(\boldsymbol{F}^T\boldsymbol{F})^{-1}$, which is the ordinary least squares solution.

On the other hand, if the prior is a Dirac pdf, the prior variance will be zero, and the posterior equals the prior. The observations will not alter the parameters.

Posterior distribution for C_i^2

As $Z(x)$-function, use

$$Z_{si}(x)|\boldsymbol{\Theta} = Z_{iB}(x)|\boldsymbol{\Theta} + Z_{iT}(x)|\boldsymbol{\Theta} \sim N(2C_i^2, 2\rho(x_1' - x_1'')(\sigma^2 + K)).$$

Let $B_1 = C_i^2,\ f^1(x) = 2,\ B_1 = f^i(x) = 0, i = 2, 3, \ldots$. Assume that shale number i is penetrated by m_i wells. The observation vector

$$\boldsymbol{Z}_{si} = \left[\begin{array}{l} z_{iB}(w_{1i}) + z_{iT}(w_{1i}) \\ \vdots \\ z_{iB}(w_{m_i i}) + z_{iT}(w_{m_i i}) \end{array} \right]$$

and the vector $F = [2, \ldots, 2]^T$ are both of length m_i. The matrix $\boldsymbol{\Sigma}$ will also in this case be a scalar $\sigma^2_{C_i^2}$ because there is only one parameter, and the matrix $\boldsymbol{C}_{kj} = 2\rho(w_{ki} - w_{ji})(\sigma^2 + \tau)$. Using expression (15) one gets:

$$E(C_i^2 | \boldsymbol{Z}_{si}) = \mu_{C_i^2} + \sigma^2_{C_i^2} \boldsymbol{F}^T (\boldsymbol{C} + \sigma^2_{C_i^2} \boldsymbol{F}\boldsymbol{F}^T)^{-1} (\boldsymbol{Z}_{si} - \boldsymbol{F}\mu_{C_i^2}) \tag{19}$$

Using expression (16) one gets:

$$Var(C_i^2 | \boldsymbol{Z}_{si}) = \sigma^2_{C_i^2} - \sigma^2_{C_i^2} \boldsymbol{F}^T (\boldsymbol{C} + \sigma^2_{C_i^2} \boldsymbol{F}\boldsymbol{F}^T)^{-1} \sigma^2_{C_i^2} \boldsymbol{F} \tag{20}$$