## Abstract

For a locally Lipschitz function $f$, it is a well-known result that the ordinary differential equation

$$\frac{dy}{dt} = f(y) \tag{1}$$

admits a unique local and stable solution.

When the function $f$ is continuous but not Lipchitz, the classical counter-example

$$\frac{dy}{dt} = \sqrt{y} \tag{2}$$

shows that uniqueness and stability are no longer guaranteed, while existence can be proved by a compactness argument.

However, one can show that the ordinary differential equation

$$\frac{dy}{dt} = \begin{cases} 1 & \text{if } y < 0 \\ 0 & \text{if } y > 0 \end{cases} \tag{3}$$

admits a unique solution which is stable with respect to the initials values.

This last example shows that the uniqueness and stability of the solutions of (1) are not directly related to the regularity of $f$. The condition that makes (3) into a well-posed problem, while (2) is not, is the so-called *tranversality condition* introduced in [1] by Bressan. The tranversality condition basically says that the solution $y(t)$ approaches the region where $f$ is discontinuous with a strictly positive angle of attack. When $f$ contains several points of discontinuity (possibly infinitly many) an efficient tool also introduced in [1] to prove well-posedness is the directional total variation of a function.

In this talk/lecture, I will present the proof given by Bressan in [2] of the well-posedness of the ordinary differential equation (1) for a function $f$ with (locally) bounded directional total variation.


## References


[1] A. Bressan. Unique solutions for a class of discontinuous differential equations. *Proc. Amer. Math. Soc.*, 104(3):772–778, 1988.

[2] A. Bressan and W. Shen. Unique solutions of discontinuous O.D.E.'s in Banach spaces. *Anal. Appl. (Singap.)*, 4(3):247–262, 2006.