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Some problems of nonlinear solitary waves interaction in nonintegrable systems

Abstract: In this report we discuss a new method for studying interaction of solitary waves such as kinks and solitons. These objects appear as solutions of nonlinear equations with a small parameter ε at the highest-order derivatives (equations with small viscosity, small dispersion, etc.). Although these exact solutions are different, all of them are approximations as $\varepsilon \to 0$ of generalized functions of $\varepsilon\delta$ type, where δ is the Dirac delta function, and of Heaviside function type. In several papers, a new technique was developed , which allows one to present smooth functions of linear combinations of these generalized functions in the form of linear combinations of these generalized functions themselves up to quantities small in the weak sense. For example, for $a, b, c, d = \text{const}$ we have $f(a+bH_\varepsilon(x)+cH_\varepsilon(x+d)) = f(a)+B(d/\varepsilon)H(x)+(1-B(d/\varepsilon))H(x+d)+O_{D'}(\varepsilon)$. Here $H_\varepsilon(x)$ is an approximation of the Heaviside function $H(x)$, $B(z) \in C^\infty$, and $B'(z) \in \mathbb{S}(\mathbb{R}^1)$.

In fact, such formulas mean that the principle of nonlinear superposition can be written explicitly for nonintegrable systems. In turn, this permits describing the interaction of above-mentioned solitary nonlinear waves by explicit formulas up to terms small in the weak sense. All this is closely connected to the notion of limiting problems for the problems containing a small parameter and to the definitions of a generalized solutions of these problems admitting the limit passage.

In the report, we discuss applications of this method to problems of interaction of solitons [1,2] in the nonintegrable case, shock waves [2], and δ-shock waves [4], as well as formation of shock waves [3] and δ-shock waves [5].

References

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