NTNU
Norwegian University of
Science and Technology

R-INLA: Practicing with examples

Andrea Riebler <andrea.riebler@math.ntnu.no>
Department of Mathematical Sciences, NTNU
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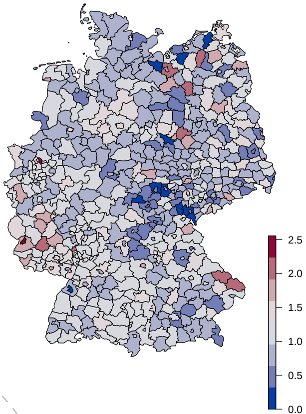
Outline

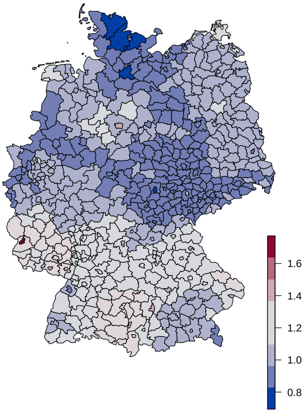
Disease mapping: Model choice

Measurement error: Joint models and copy feature

Disease mapping in Germany

We observed larynx cancer mortality counts for males in 544 district of Germany from 1986 to 1990 as well as level of smoking consumption (100 possible values).





The data

y_i: The cancer mortality count at location i.

E_i: An offset; expected number of cases in district i.

c_i: A covariate (level of smoking consumption) at location i

$\boldsymbol{s}_i$: spatial location i (here, district).

Here $i = 1, \ldots, 544$.

The model

$$y_i \mid \eta_i \sim \text{Poisson}(E_i \exp(\eta_i))$$
$$\eta_i = \mu + f(c_i) + f_s(s_i) + \underbrace{u_i}_{f_u(s_i)}$$

— $f(c_i)$ is as smooth effect of the covariate $\boldsymbol{c}$

$$\boldsymbol{f} = \{f_1, \ldots, f_{100}\} \sim \text{RW2}(\tau_c)$$

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$$f_s(s) \mid f_s(s'), s \neq s', \tau_s \sim \mathcal{N}\left(\frac{1}{n_s} \sum_{s \sim s'} f_s(s'), \frac{\tau_s}{n_s}\right)$$

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— $f_u(s_i)$ is an unstructured random effect

$$f_s(s) \mid \boldsymbol{f}_u = \{f_u(s_1), \ldots, f_u(s_{544})\} \sim \mathcal{N}(0, \tau_u \mathbf{I})$$

Constraints

For identifiability of the intercept, a sum-to-zero constraint is default for all intrinsic models, so

$$\sum_j f_s(s_j) = 0$$

$$\sum_i f_i = 0.$$

Exercise: Model fitting and comparison

Fit the following models

1. The standard BYM model, i.e. only including μ, f_s, f_u.
2. BYM + linear covariate
3. BYM + non-linear covariate

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Compare the models using

— The deviance information criterion

— The log-score: LS $= -\sum_i \log(\text{CPO}_i)$

The classical measurement error model

Linear predictor: $\eta_i = \beta_0 + \beta_x x_i + \boldsymbol{\beta}_z^\top \boldsymbol{z}_i$.

Here, $\boldsymbol{x}$ is the correct but *unobserved* covariate and $\boldsymbol{w}$ the observed variable with error. Then

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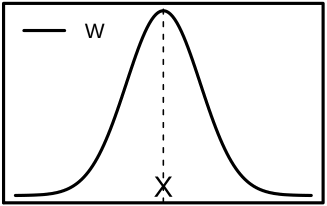
$$\begin{aligned} \boldsymbol{w} &= \boldsymbol{x} + \boldsymbol{u} \\ \boldsymbol{u} &\sim \mathcal{N}(0, \tau_u \mathbf{D}) \ , \end{aligned}$$

is the classical ME model.



Here, $\mathbf{D} = \text{diag}(d_1, \dots, d_n)$ and $d_i \propto \tau_{u_i} = \tau_u(x_i)$.

Assumption: $\boldsymbol{u}$ is independent of $\boldsymbol{x}$, but also of $\boldsymbol{z}$ and $\boldsymbol{y}$.

Easy example

Consider the following setting:

$$\boldsymbol{y} \sim \mathcal{N}(\beta_0 + \beta_x \boldsymbol{x}, \tau_y)$$

$$\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{x}, \tau_u)$$

Goal:

Simulate data, let $\boldsymbol{x} \sim \mathcal{N}(0, 1)$, and estimate β_x back. We will try different strategies.

Exercise: Naive model specification

1. Fit the model using the observed covariate $\boldsymbol{w}$ and compare the estimated effect β_x to what we would get if we knew the correct covariate $\boldsymbol{x}$. What do you observe?

Exercise: Plug-in approach

2. Use a plug-in approach:
   a) Set up a model

   $$\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{x}, \tau_u)$$
   $$\boldsymbol{x} \sim \mathcal{N}(0, \tau_x)$$

   where $\tau_x = 1$ is assumed to be known.

   b) Estimate

   $$\boldsymbol{y} \sim \mathcal{N}(\hat{\boldsymbol{x}}, \tau_y)$$

   where $\hat{\boldsymbol{x}}$ is the posterior mean obtained in a).

Exercise: Joint model

3. Use a joint model to incorporate also the uncertainty of $\hat{\boldsymbol{x}}$.

$$\begin{pmatrix} \boldsymbol{y} & \mathtt{NA} \\ \mathtt{NA} & \boldsymbol{w} \end{pmatrix} = \begin{pmatrix} \mu \\ \mathtt{NA} \end{pmatrix} + \begin{pmatrix} \beta_x \boldsymbol{x} \\ \mathtt{NA} \end{pmatrix} + \begin{pmatrix} \mathtt{NA} \\ \boldsymbol{x} \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{\tau_y}} \epsilon \\ \mathtt{NA} \end{pmatrix} + \begin{pmatrix} \mathtt{NA} \\ \frac{1}{\sqrt{\tau_u}} \widetilde{\epsilon} \end{pmatrix}$$

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Question: Is this of interest in real applications.

Framingham heart study

- Data from the Framingham heart study 1973-1986, aiming to understand the factors leading to coronary heart disease $\boldsymbol{y} \in \{0, 1\}$ in individuals.
- Covariates: $\boldsymbol{x} = \log(\text{SBP} - 50)$ and smoking status $\boldsymbol{z} \in \{0, 1\}$.
- $\boldsymbol{x}$ could be measured only with error at two different exams. Values were $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ such that

$$w_{ij} \mid x_i \sim \mathcal{N}(x_i, \tau_u).$$

- Exposure model:

$$\boldsymbol{x} \mid \boldsymbol{z} \sim \mathcal{N}(\alpha_0 + \alpha_z \boldsymbol{z}, \tau_x \mathbf{I})$$

- Regression model:

$$\boldsymbol{\eta} = \text{logit}[P(\boldsymbol{y} = 1 \mid \boldsymbol{x}, \boldsymbol{z})] = \beta_0 + \beta_x \boldsymbol{x} + \beta_z \boldsymbol{z}.$$

How does this look in INLA notation

$$
\underbrace{\begin{bmatrix} y_1 & \texttt{NA} & \texttt{NA} \\ \vdots & \vdots & \vdots \\ y_n & \texttt{NA} & \texttt{NA} \\ \texttt{NA} & 0 & \texttt{NA} \\ \vdots & \vdots & \vdots \\ \texttt{NA} & 0 & \texttt{NA} \\ \texttt{NA} & \texttt{NA} & w_{11} \\ \vdots & \vdots & \vdots \\ \texttt{NA} & \texttt{NA} & w_{1n} \\ \texttt{NA} & \texttt{NA} & w_{21} \\ \vdots & \vdots & \vdots \\ \texttt{NA} & \texttt{NA} & w_{2n} \end{bmatrix}}_{\texttt{Y}} = \beta_0 \underbrace{\begin{bmatrix} 1 \\ \vdots \\ 1 \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \end{bmatrix}}_{\texttt{beta.0}} + \beta_x \underbrace{\begin{bmatrix} 1 \\ \vdots \\ n \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \end{bmatrix}}_{\texttt{beta.x}} + \underbrace{\begin{bmatrix} \texttt{NA} \\ \vdots \\ \texttt{NA} \\ -1 \\ \vdots \\ -n \\ 1 \\ \vdots \\ n \\ 1 \\ \vdots \\ n \end{bmatrix}}_{\texttt{idx.x}} + \beta_z \underbrace{\begin{bmatrix} z_1 \\ \vdots \\ z_n \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \end{bmatrix}}_{\texttt{beta.z}} + \alpha_0 \underbrace{\begin{bmatrix} \texttt{NA} \\ \vdots \\ \texttt{NA} \\ 1 \\ \vdots \\ 1 \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \end{bmatrix}}_{\texttt{alpha.0}} + \alpha_z \underbrace{\begin{bmatrix} \texttt{NA} \\ \vdots \\ \texttt{NA} \\ z_1 \\ \vdots \\ z_n \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \\ \texttt{NA} \\ \vdots \\ \texttt{NA} \end{bmatrix}}_{\texttt{alpha.z}} .
$$

Model formulation

Caution: In measurement error models prior distributions are very important. Here, they are elicited from expert/prior knowledge.

For detailed model formulation, see Example 5.2 in

`http://www.r-inla.org/examples/case-studies/muff-etal-2014`