Mid-term examination in MA0001 Mathematical methods A
Wednesday 8 October 2003 10.15–12.00

Permitted aids: Any written and printed material. Calculator.

Mark one answer for each problem on the form overleaf. You will score one point for each right answer and zero points for each wrong answer. Multiple answers will score zero.

Problem 1. A cubical block of ice with edge 20 cm begins to melt at 8 a.m. Each edge decreases steadily thereafter, and the block keeps its cubical form. At 4 p.m. the edges are 8 cm. What was the instantaneous rate of change of the volume of the block of ice at 12 noon?

(a) -882 cm^3/hour   (b) -588 cm^3/hour   (c) -3.375 cm^3/hour

Problem 2. What is $\lim_{x\to\infty} \frac{3x^3-x}{2x^3+7x^2-4}$ equal to?

(a) 1.5   (b) 0   (c) ∞

Problem 3. Let f be defined by $f(x) = \ln(x-2)$ for all $x > 2$. Let g be the inverse function of f. Which is true for all real numbers x?

(a) $g(x) = e^{x+2}$ (b) $g(x) = e^x + 2$ (c) $g(x) = e^{x-2}$

Problem 4. What is $\frac{d}{dx} \tan(2x^3)$ equal to?

(a) $6x^2/\cos^2(2x^3)$ (b) $\tan(6x^2)/\cos^2(2x^3)$ (c) $1/\cos^2(6x^2)$

Problem 5. What is $\frac{d}{dx}(5e^x)$ equal to?

(a) $5xe^{x-1}$ (b) $5e^x$ (c) 0

Problem 6. Which formula is correct for all a and b?

(a) $3^{a+b} = 3^a 3^b$ (b) $(a+b)^3 = a^3 + b^3$ (c) $\ln(a+b) = \ln a + \ln b$

Problem 7. What is $\lim_{x\to 0} \left(\frac{1}{\sin x} \frac{1-\cos x}{x}\right)$ equal to?

(a) 0   (b) 1/2   (c) ∞

Problem 8. In the sequence $\{a_n\}$, $a_n = 2n/(5n-3)$ for all positive integers n. To what does the sequence converge (or diverge)?

(a) ∞ (b) 0   (c) 2/5

Problem 9. Assume that $f(x) = \frac{x-2}{x^2-4}$ for all $x \neq -2$ and $x \neq 2$. Which number must $f(2)$ equal for f to be continuous at 2?

(a) 2   (b) 0   (c) 1/4

Problem 10. What is $\lim_{x\to 0} \frac{\sqrt{x+25}-5}{x}$ equal to?

(a) 0.2   (b) 0.1   (c) ∞

Problem 11. An animal population has size $N(t)$ at time t. The derivative of the population size satisfies the equation $\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right)$, where r and K are positive numbers. For which population size is the population's instantaneous growth rate maximal?

(a) The growth rate is larger the larger $N(t)$ is   (b) $K/2$ (c) K

Problem 12. The function f is defined by $f(x) = \frac{1}{x+1} - \frac{1}{x-1}$ for all x not equal to -1 and 1. What is $f'(x)$ equal to?

(a) $\frac{4x}{(x^2-1)^2}$ (b) $\frac{1}{(x+1)^2} - \frac{1}{(x-1)^2}$ (c) $-\frac{1}{(x+1)^2} - \frac{1}{(x-1)^2}$

Problem 13. The function f is defined by $f(x) = (2x^2 - 1)(x^3 + 2)$ for all x. What is $f'(x)$ equal to?

(a) $10x^4 - 3x^2 + 8x$ (b) $12x^3$ (c) $10x^4 - 12x^3 - 3x^2 + 8x$

Problem 14. What is the equation of the tangent line of the graph of the equation $x^3 + y^3 = 3xy$ at the point $(\frac{2}{3}, \frac{4}{3})$?

(a) $12x + 15y - 28 = 0$ (b) $4x - 5y + 4 = 0$ (c) $12x - 9y + 4 = 0$

Problem 15. The half-life of a radioactive material that decays exponentially is 5700 years. Approximately how long time will it take for 37 % of the material to decay?

(a) 2100 years   (b) 3800 years   (c) 4200 years

Oppgave	a	b	c
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

Studentnummer   Student number

Studieprogram   Study program

Inspektør   Inspector