Examination in MA0001 Mathematical methods A—Appendix Thirsday 9 June 2005

Permitted aids: Any written and printed material. One calculator.

Mark one answer for each problem on the form overleaf. You will score one point for each right answer and zero points for each wrong answer. Multiple answers will score zero.

Note: There is text on both sides of the sheet. All problems have five alternative answers.

Problem 1. Find the limit

$$\lim_{x \to \infty} \frac{\sqrt{x^2 - 1}}{\sqrt{4x^2 - x}}.$$

(a) 0.5 (b) 0 (c) 0.25 (d) ∞ (e) The limit does not exist

Problem 2. The yield (in mg) of a chemical reaction is $8x/(1+4x^2)$, where x is the concentration (in M) of a substance. What is the largest possible yield?

(a) 8 mg (b) 4 mg (c) 2 mg (d) 7 mg (e) 1 mg

Problem 3. Find the limit

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right).$$

(a) 1/2 (b) The limit does not exist (c) 0 (d) 1 (e) -1/2

Problem 4. Which formula is correct for all a and b?

(a)
$$(2^a)^b = 2^{a+b}$$
 (b) $(2^a)^b = 2^{a^b}$ (c) $(2^a)^b = 2^{a+\ln b}$ (d) $(2^a)^b = b \cdot 2^a$ (e) $(2^a)^b = 2^{ab}$

Problem 5. A function f is defined by $f(x) = \sqrt[3]{x/5} - 2$ for all x. Let f^{-1} be the inverse function of f. What is $f^{-1}(-1)$ equal to?

(a) 5 (b)
$$-\sqrt[3]{3}$$
 (c) 1 (d) -27 (e) $-\sqrt[3]{7}$

Problem 6. A function f is defined by $f(x) = \int_0^{x^2} e^{3t^2} dt$. Find f'(x).

(a)
$$2xe^{3x^4}$$
 (b) $4x^3e^{3x^4}$ (c) $e^{3x^4}/3$ (d) $(4x+24x^5)e^{3x^4}$ (e) e^{3x^4}

Problem 7. An animal population has size $1000 + 100te^{-t/50}$, where t is the number of years passed since 1950. Approximately what is the maximum population size?

(a) 2800 (b) 1100 (c) 2000 (d) The population grows beyond all limits (e) 1200

Problem 8. Evaluate the integral $\int_1^\infty \frac{dx}{x(2+\ln x)^2}$.

(a) 1/4 (b) 0 (c) 1/8 (d) The integral diverges (e) 1/2

Problem 9. Evaluate the integral $\int \frac{dx}{x^2-1}$.

(a)
$$-10/(2x+1)^2 - 5/(x-2)^2 + C$$

(b) $\ln(x^2 - 1) + C$

- (c) -1/(x-1) + C
- (d) $\frac{1}{2} \ln |(x-1)/(x+1)| + C$

(e)
$$-x - 1/x + C$$

Problem 10. Where does the function f defined by $f(x) = x^3 - 6x^2$ have an inflection point?

(a) 6 (b) 4 (c) -2 (d) 0 (e) 2

Problem 11. Americium-241, which is used in smoke detectors, has a half-life of 458 years. Approximately how long will it take for 10% of a sample to decay?

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(a) 62 years (b) 92 years (c) 46 years (d) 102 years (e) 70 years

Problem 12. A function f is defined by $f(t) = \ln(2e^{3t})$ for all t. Find f'(t).

(a) $e^{-3t}/2$ (b) $3/(2+e^{-3t})$ (c) $1/2+e^{-3t}$ (d) 3 (e) $1/(2e^{3t})$

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