Mid-term examination in MA0001 Mathematical methods A
Wednesday 13 October 2004 10:15–12:00

Permitted aids: Any written and printed material. One calculator.

Mark one answer for each problem on the form overleaf. You will score one point for each right answer and zero points for each wrong answer. Multiple answers will score zero.

NB: There is text on both sides of the sheet.

Problem 1. Find $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.

(a) ∞ **(b)** 0 **(c)** The limit doesn't exist **(d)** 1/2

Problem 2. A function f is defined by $f(x) = \frac{2x+3}{x-1}$ for all x different from 1. Find $f'(x)$.

(a) $-\frac{5}{(x-1)^2}$ **(b)** 2 **(c)** $\frac{4x+1}{(x-1)^2}$ **(d)** f isn't differentiable

Problem 3. Which formula is correct for all positive numbers a and b?

(a) $\ln(ab) = \ln a + \ln b$ **(b)** $\ln a^b = b + \ln a$ **(c)** $4^{a-b} = 4^a - 4^b$ **(d)** $(a-b)^2 = a^2 - b^2$

Problem 4. Find $\frac{d}{dx} x^{x+2}$. (Hint: Logarithmic differentiation may be applied.)

(a) $(x+2)x^{x+1}$ **(b)** $x^{x+2}\ln x$ **(c)** $\left(\frac{x}{x+2} + \ln(x+2)\right) x^{x+2}$ **(d)** $\left(1 + \frac{2}{x} + \ln x\right) x^{x+2}$

Problem 5. Find $\frac{d}{dx} e^{5x}$.

(a) $5e^{5x-1}$ **(b)** $5e^{5x}$ **(c)** $e^{5x} + e^5$ **(d)** $5xe^{5x-1}$

Problem 6. Find $\lim_{x\to-\infty} \frac{7x^3-x}{5x^3-8x-1}$.

(a) ∞ **(b)** 1.4 **(c)** $-\infty$ **(d)** 0

Problem 7. Find $\frac{d}{dx}\sin(3x^2)$.

(a) $6x\cos(3x^2)$ **(b)** $\cos(6x)$ **(c)** $\cos(3x^2) + \sin(6x)$ **(d)** $6\cos x$

Problem 8. Find $\lim_{x\to 0} \frac{x}{\sqrt{x+9}-3}$.

(a) ∞ **(b)** 6 **(c)** 3 **(d)** 0

Problem 9. I an animal population the proportion of a deleterious gene is $q_0 = 0.3$. After n generations the proportion is q_n, which is given recursively by $q_n = 1 - \frac{0.9999}{1+q_{n-1}}$, $n \geq 1$. What is the proportion of the deleterious gene in the long run?

(a) 1 **(b)** 0.01 **(c)** 0.02 **(d)** 0

Problem 10. The half-life of a radioactive material that decays exponentially is 2000 years. Approximately how long will it take for 30 % of the material to decay?

(a) 800 years **(b)** 1000 years **(c)** 600 years **(d)** 1200 years

Problem 11. Let f be defined by $f(x) = 3e^{3x}$ for all real numbers x. Let f^{-1} be the inverse function of f. Find $f^{-1}(x)$.

(a) $\ln x - \frac{1}{3}\ln 3$ **(b)** f doesn't have an inverse function **(c)** $\frac{1}{3}(\ln x - \ln 3)$ **(d)** $\frac{1}{3}\ln x - \ln 3$

Problem 12. A boat sails parallel to a straight beach at a constant speed of 12 knots (12 nautical miles per hour), staying 4 nautical miles offshore. How fast is it approaching a lighthouse on the shoreline at the instant it is exactly 5 nautical miles from the lighthouse?

(a) 9.6 knots **(b)** 9 knots **(c)** 7.2 knots **(d)** 20 knots

Problem	a	b	c	d
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Studentnummer Student number

Studieprogram Study program

Inspektør Inspector