Norwegian University of Science and Technology
Department of Mathematical Sciences

English

Contact during examination: Øyvind Bakke
Telephone: 73 59 81 26, 990 41 673

ST0201 Statistics with Applications

Saturday 2 June 2007
9:00–13:00
Permitted aids: Any written and printed material. One calculator.
Grades to be announced: 23 June 2007

The final examination consists of two parts:

1. The problems on the next page.
2. Appendix with a multiple choice questionnaire.

The Appendix is to be submitted with the form filled in together with the answer to part (1). Part (1) and (2) count equally in the evaluation of the final examination.

In addition to the final examination the mid-term examination counts 20% if it is advantageous to the candidate.

In the evaluation of part (1) (next page) each of the six points counts equally.

In part (1) you should demonstrate how you arrive at your answers (e.g. by including intermediate answers or referral to theory). Answers based on calculator or table look-up only will not be accepted.

Problem 1

The Botanical Research Station investigated how various kinds of fertilization affect the growth of sunflowers. Two kinds of fertilization were applied. Four randomly selected sunflowers were fertilized in one way, method A, and were x_i cm taller in a week, while four other randomly selected sunflowers were fertilized in another way, method B, and were y_i cm taller. Assume that x_i and y_i are independent observations from two normal distributions with the same variance. The growth gain of the plants were:

x_i (method A)	12.0	10.2	12.1	10.4
y_i (method B)	13.0	11.0	12.0	11.7

It is given that $\bar{x} = 11.175$, $\sum(x_i - \bar{x})^2 = 3.0875$, $\bar{y} = 11.925$ and $\sum(y_i - \bar{y})^2 = 2.0675$.

a) Perform a test to investigate whether method B gives a larger expected growth gain than method A. The null hypothesis is that A gives a growth gain that is at least as large as B gives. Use significance level $\alpha = 0.05$.

b) Find a 95% confidence interval for the difference between the expected growth gains of the two fertilization methods.

c) In addition, a third fertilization method was investigated. Four randomly selected sunflowers gained the following growth in a week:

z_i (method C)	13.5	12.5	12.5	12.7

It is given that at $\bar{z} = 12.8$ and $\sum(z_i - \bar{z})^2 = 0.68$. Perform one test to investigate whether there are differences in the expected growth gains with the three fertilization methods. (You are use one test, not test the methods pairwise against each other.) Use significance level $\alpha = 0.05$. It is given that $P(F > 4.26) = 0.05$ if F is F-distributed with 2 and 9 degrees of freedom.

d) The three fertilization methods consisted of giving 1, 2 and 3 grams of fertilizer per litre of water. We now assume a linear regression model with 12 observations, were x_i is now the amount of fertilizer (g/dm^3) and y_i growth gain (cm):

x_i	1	1	1	1	2	2	2	2	3	3	3	3
y_i	12.0	10.2	12.1	10.4	13.0	11.0	12.0	11.7	13.5	12.5	12.5	12.7

It is given that $\bar{x} = 2$, $\bar{y} = 11.967$, $\sum(x_i - \bar{x})^2 = 8$, $\sum(y_i - \bar{y})^2 = 11.127$ and $\sum(x_i - \bar{x})(y_i - \bar{y}) = 6.5$. Estimate the regression line (find $\hat{\alpha}$ and $\hat{\beta}$). Perform a test to investigate whether the expected growth gain depends on the amount of fertilizer (i.e., investigate whether the slope β is equal to zero or not). Use significance level 0.05.

Problem 2

An animal species has 0, 1 or 2 offspring each year. A biologist examined 55 randomly selected animals and found that 27 of them had 0 offspring, 22 had 1 offspring and 6 had 2 offspring.

a) Perform a test to investigate whether the number of offspring, X, comes from the probability distribution given by $P(X = x) = 0.4^x/1.56, \quad x = 0, 1, 2$. Use significance level 0.05.

b) Assume that the probability distribution has the form $P(X = x) = c^x/(1 + c + c^2)$, $x = 0, 1, 2$, where c is a positive parameter. (Then $c = 0.4$ gives the distribution from (a).) Find the maximum likelihood estimate of c. (You are not required to prove that the critical point in fact gives a maximum.)