Mid-term examination in ST0201 Statistics with Applications
Tuesday 4 March 2008 15:15–17:00

Permitted aids: Any written and printed material. One calculator.

Mark one answer for each problem on the form overleaf. You will score one point for each right answer and zero points for each wrong answer. Multiple answers will score zero.

NB: There is text on both sides of the sheet. All problems have five alternative answers.

Problem 1. We take 36 samples of a rock of unknown age μ and assume that the results of the age determination are normally distributed with expected value (mean) μ (million years) and known standard deviation 3.6 (million years). We shall test $H_0\colon \mu = 68$ against the alternative $H_1\colon \mu \neq 68$. We let the rejection area of the test be the union of $\bar{x} < 67$ and $\bar{x} > 69$, where $\bar{x}$ is the average age of the samples (in million years). What is the probability of making a type I error, if the age is indeed 68 million years?

   (a) 0.095   **(b)** 0.055   **(c)** 0.085   **(d)** 0.065   **(e)** 0.075

Problem 2. We perform the test from the previous problem with 36 (new) samples and with the same rejection area. What is the probability that H_0 will be rejected if μ is indeed 68.5?

   (a) 0.42   **(b)** 0.78   **(c)** 0.93   **(d)** 0.53   **(e)** 0.21

Problem 3. A chemical reaction was performed five times. The yields were 43.7, 38.9, 42.9, 37.3 and 45.4 mg. Find an estimate of the expected yield of this reaction.

   (a) 40.2 mg   **(b)** 41.6 mg   **(c)** 42.4 mg   **(d)** 39.6 mg   **(e)** 43.2 mg

Problem 4. Assume that the yield of the chemical reaction of the previous problem is normally distributed. Find a 95% confidence interval for the expected yield (in mg).

   (a) $[37.4, 45.9]$ **(b)** $[36.9, 46.4]$ **(c)** $[38.4, 44.9]$ **(d)** $[38.9, 44.5]$ **(e)** $[37.9, 45.4]$

Problem 5. X and Y are independent and normally distributed with expected value (mean) μ and standard deviation σ. Find $P\left(\left(\frac{X-\mu}{\sigma}\right)^2 + \left(\frac{Y-\mu}{\sigma}\right)^2 > 7.38 \right)$.

   (a) 0.025   **(b)** 0.005   **(c)** 0.010   **(d)** 0.050   **(e)** 0.100

Problem 6. Assume that X is t distributed with 13 degrees of freedom. Find a number a such that $P(X < a) = 0.025$.

   (a) -2.18 **(b)** -2.16 **(c)** -2.18 **(d)** 2.16 **(e)** 2.145

Problem 7. X and Y are independent and exponentially distributed with parameter λ. Find a number c such that $P(2\lambda(X + Y) < c) = 0.05$.

   (a) 0.71   **(b)** 0.10   **(c)** 9.49   **(d)** 5.99   **(e)** 12.48

Problem 8. The number of offspring in a brood of an animal species is assumed to be geometrically distributed with parameter p. A biologist examines 73 broods, and find 253 offspring in total in the broods. Find the maximum likelihood estimate of p.

   (a) 0.59   **(b)** 0.19   **(c)** 0.49   **(d)** 0.39   **(e)** 0.29

Problem 9. We examine how much, Y (grams), of a substance that can be dissolved in 100 g of water at various temperatures X (°C). We perform an experiment with 18 samples, and get an empirical covariance of $S_{XY} = 4048$ between X and Y and a sample variance of $S_X^2 = 695$ for X. Find an estimate of the slope in a linear regression model for Y against X ($EY = \alpha + \beta X$).

    (a) 7.33   **(b)** 5.82   **(c)** 3.66   **(d)** 6.34   **(e)** 4.52

Problem 10. What is the probability that a t distributed random variable with 15 degrees of freedom is greater than 2.131?

    (a) 0.95   **(b)** 0.01   **(c)** 0.05   **(d)** 0.5   **(e)** 0.025

Problem	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Studentnummer   Student number

Studieprogram   Study program

Inspektør   Inspector