

Exercise on Estimating Equations

Estimating Equations for Moment Estimators

Suppose $\boldsymbol{\theta} = (\theta_1, \theta_2)$ and that Z under $\boldsymbol{\theta}$ has the density $f(z; \boldsymbol{\theta})$. Define the functions μ_1 and μ_2 by

$$\begin{aligned}E_{\boldsymbol{\theta}} Z &= \mu_1(\theta_1, \theta_2) \\E_{\boldsymbol{\theta}} Z^2 &= \mu_2(\theta_1, \theta_2)\end{aligned}$$

Let Z_1, Z_2, \dots, Z_n be independent observations of Z .

What is meant by the *moment estimators* $(\hat{\theta}_1, \hat{\theta}_2)$ of (θ_1, θ_2) ?

Show that the moment estimators are obtained from the estimating equations corresponding to

$$\eta(z, \boldsymbol{\theta}) = (z - \mu_1(\theta_1, \theta_2), z^2 - \mu_2(\theta_1, \theta_2))$$

How can this be used to obtain the asymptotic distribution of $(\hat{\theta}_1, \hat{\theta}_2)$?