Solution

Ex. 2.7 b - d

b) $E_{y|x}\left(f(x_0) - \hat{f}(x_0)\right)^2 = Var_{y|x}\left(\hat{f}(x_0)\right)$

$+\left[E_{y|x}\left(\hat{f}(x_0) - f(x_0)\right)\right]^2$

$= \sigma^2 \sum_{i=1}^{N} l_i^2(x_0; \mathcal{X}) f(x_i) + \left(\sum_{i=1}^{N} l_i(x_0; \mathcal{X}) f(x_i) - f(x_0)\right)^2$

(using $E()^2 = Var() + (E())^2$)

c) $E_{y,x}\left(f(x_0) - \hat{f}(x_0)\right)^2$

$= Var_{y,x}\left(f(x_0) - \hat{f}(x_0)\right)$ ← a constant

$+\left[E_{y,x}\left(f(x_0) - \hat{f}(x_0)\right)\right]^2$

$= Var_{x}\left[E_{y|x}\hat{f}(x_0)\right]$

$+ E_x\left[Var_{y|x}\hat{f}(x_0)\right]$ (using hint)

$+\left(f(x_0) - E_{y,x}\hat{f}(x_0)\right)^2$

$= Var_x\left[\sum_{i=1}^{N} l_i(x_0; \mathcal{X}) f(x_i)\right]$

$+ E_x\left[\sigma^2 \sum_{i=1}^{N} l_i^2(x_0; \mathcal{X}) f(x_i)\right] + \left[\sum_{i=1}^{N} E_x l_i(x_0; \mathcal{X}) f(x_i) - f(x_0)\right]^2$

Thus we have essentially one more term in the last expression; $Var_x\left[E_{y|x}\hat{f}(x_0)\right]$