

4.2

2.18. Denote by $F(x)$ the distribution function of X . Suppose that $mX < c$. Then

$$\begin{aligned} E|X - c| &= \int_{-\infty}^{\infty} |x - c| dF(x) = \int_{-\infty}^{mX} |x - c| dF(x) + \int_{mX}^c |x - c| dF(x) + \int_c^{\infty} |x - c| dF(x) \\ &= c \int_{-\infty}^{mX} dF(x) - \int_{-\infty}^{mX} xdF(x) + c \int_{mX}^c dF(x) - \int_{mX}^c xdF(x) + \int_c^{\infty} xdF(x) - c \int_c^{\infty} dF(x) \end{aligned}$$

and

$$\begin{aligned} E|X - mX| &= mX \int_{-\infty}^{mX} dF(x) - \int_{-\infty}^{mX} xdF(x) - mX \int_{mX}^c dF(x) + \int_{mX}^c xdF(x) \\ &\quad + \int_c^{\infty} xdF(x) - mX \int_c^{\infty} dF(x), \end{aligned}$$

hence

$$\begin{aligned} &E|X - c| - E|X - mX| \\ &= (c - mX) \int_{-\infty}^{mX} dF(x) - (c - mX) \int_c^{\infty} dF(x) - (c - mX) \int_{mX}^c dF(x) + 2 \int_{mX}^c (c - x) dF(x) \\ &= (c - mX)(P(X \leq mX) - P(X > mX)) + 2 \int_{mX}^c (c - x) dF(x) \geq 0. \end{aligned}$$

In case $mX > c$, the proof is analogous.