MNFST102 Oppgaver (transformasjoner)

## EXERCISES

**2.1** In each of the following find the pdf of $Y$ and the range of $Y$. Show that the pdf integrates to 1.

a. $f_X(x) = 42x^5(1-x)$, $0 < x < 1$; $Y = X^3$

b. $f_X(x) = 7e^{-7x}$, $0 < x < \infty$; $Y = 4X + 3$

**2.2** In each of the following find the pdf of $Y$ and the range of $Y$

a. $f_X(x) = 1$, $0 < x < 1$; $Y = X^2$

b. $X$ has pdf

$$f_X(x) = \frac{(n+m+1)!}{n!m!} x^n (1-x)^m, \quad 0 < x < 1, \quad m, n \text{ positive integers};$$

$$Y = -\log X$$

c. $X$ has pdf

$$f_X(x) = \frac{1}{\sigma^2} x e^{-(x/\sigma)^2/2}, \quad 0 < x < \infty, \quad \sigma^2 \text{ a positive constant};$$

$$Y = e^X$$

**2.3** Suppose $X$ has the geometric pmf, $f_X(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x$, $x = 0, 1, 2, \ldots$. Determine the probability distribution of $Y = X/(X+1)$. Note that here both $X$ and $Y$ are discrete random variables. To specify the probability distribution of $Y$, specify its pmf.

**2.5** Let $X$ have pdf

$$f(x) = \begin{cases} 30x^2(1-x)^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}.$$

Find the pdf of $X^2$.

**2.10** If the random variable $X$ has pdf

$$f(x) = \begin{cases} \frac{x-1}{2} & 1 < x < 3 \\ 0 & \text{otherwise} \end{cases},$$

find a monotone function $u(x)$ such that the random variable $Y = u(X)$ has a uniform(0, 1) distribution.

## EXERCISES

**4.1** A random point $(X, Y)$ is distributed uniformly on the square with vertices $(1, 1), (1, -1), (-1, 1), (-1, -1)$. That is, the joint pdf is $f(x, y) = \frac{1}{4}$ on the square. Determine the probabilities of the following events.

**4.4** A pdf is defined by

$$f(x,y) = \begin{cases} C(x+2y) & \text{if } 0 < y < 1 \text{ and } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}.$$

a. Find the value of $C$.
b. Find the marginal distribution of $X$.
c. Find the joint cdf of $X$ and $Y$.
d. Find the pdf of the random variable $Z = 9/(X+1)^2$.

**4.5** Find $P(X > \sqrt{Y})$ if $X$ and $Y$ are jointly distributed with pdf

$$f(x,y) = x + y, \quad 0 \le x \le 1, \quad 0 \le y \le 1.$$

**4.19** a. Let $X_1$ and $X_2$ be independent n(0, 1) random variables. Find the pdf of $(X_1 - X_2)^2/2$.
b. If $X_i, i = 1, 2$, are independent gamma$(\alpha_i, 1)$ random variables, find the distribution of $X_1/(X_1 + X_2)$ and $X_2/(X_1 + X_2)$.

**4.20** $X_1$ and $X_2$ are independent n$(0, \sigma^2)$ random variables.
a. Find the joint distribution of $Y_1$ and $Y_2$, where

$$Y_1 = X_1^2 + X_2^2 \quad \text{and} \quad Y_2 = \frac{X_1}{\sqrt{Y_1}}.$$

b. Show that $Y_1$ and $Y_2$ are independent, and interpret this result geometrically.