Løsningsforslag . august 2000

Oppgave 1 Eksamen levetidsanalyse C1

Distribution Analysis

Variable: C1

Censoring Information	Count
Uncensored value	5
Right censored value	1

Nonparametric Estimates

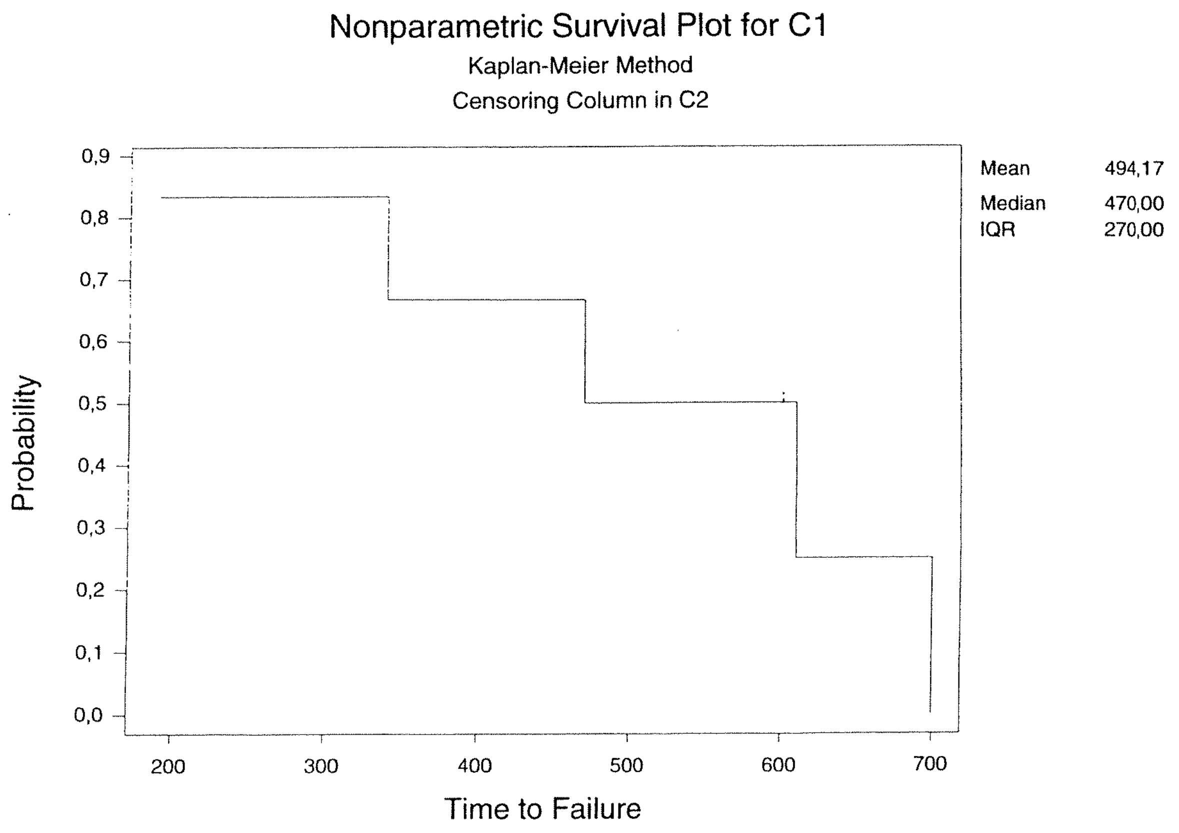
Characteristics of Variable

Mean	Standard Error	95,0% Normal CI lower	upper
494,1667	84,0765	329,3797	658,9537

Median = 470,0000
IQR = 270,0000 Q1 = 340,0000 Q3 = 610,0000

Kaplan-Meier Estimates

Time	Number at Risk	Number Failed	Survival Probability	Standard Error	95,0% Normal CI Lower	Upper
190,0000	6	1	0,8333	0,1521	0,5351	1,0000
340,0000	5	1	0,6667	0,1925	0,2895	1,0000
470,0000	4	1	0,5000	0,2041	0,0999	0,9001
610,0000	2	1	0,2500	0,2041	0,0000	0,6501
700,0000	1	1	0,0000	0,0000	0,0000	0,0000



2a

$$d_i \begin{cases} 1 & T_i \le C_i \\ 0 \end{cases} \qquad y_i = \min(T_i, C_i)$$

$$\ell(Y_1, \ldots, Y_n) = \prod_{i=1} f(Y_i)^{d_i} R(Y_i)^{1-d_i}$$

$$L = \ln(\ell) = \sum_{i=1}^{n} [d_i \ln f(Y_i) + (1-d_i) \ln R(Y)]$$

b

$$f(y_i) = \alpha \lambda^{\alpha} y_i^{\alpha-1} e^{-(\lambda y_i)^\alpha} \qquad R(y) = e^{-(\lambda y_i)^\alpha}$$

$$L = \sum_{i=1}^{n} d_i [\ln\alpha + \alpha \ln \lambda + (\alpha-1)\ln Y_i] + \sum_{i=1}^{n} (-\lambda^\alpha Y_i^\alpha)$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \left[\frac{d_i \cdot \alpha}{\lambda} - \alpha \lambda^{\alpha-1} \cdot Y_i^\alpha\right] = 0$$

$$\alpha \sum_{i=1}^{n} d_i - \alpha\lambda^\alpha \cdot \sum_{i=1}^{n} Y_i^\alpha = 0$$

$$\lambda^\alpha = \frac{\sum_i^n d_i}{\sum_{i=1}^{n} Y_i^\alpha} = \frac{d}{\sum_{i=1}^{n} Y_i^\alpha} \qquad \lambda^* = \left(\frac{d}{\sum_1^n Y_i^\alpha}\right)^{\frac{1}{\alpha}}$$

$d = \Sigma d_i$

$d = 5 \quad \alpha = 3$

$$\lambda^* = \frac{5^{1/3}}{100\,(1.9^3 + 3.4^3 + 4.7^3 + 5.4^3 + 6.1^3 + 7^3)^{1/3}}$$

$$\lambda^* = \frac{5^{1/3}}{100\,(877.4)^{1/3}}$$

0.001786

$$\lambda^* \ \boxed{\approx 0.0018}$$

$\lambda^* \approx 0.0018$

d)

$$\frac{\partial^2 L}{\partial \lambda^2} = \alpha \cdot d \cdot \left(-\frac{1}{\lambda^2}\right) - \alpha(\alpha-1)\,\lambda^{\alpha-2} \sum_{i=1}^{n} Y_i^{\alpha}$$

$$-\frac{\partial^2 L}{\partial \lambda^2} = \frac{\alpha d}{\lambda^2} + \alpha(\alpha-1)\,\lambda^{\alpha-2}\sum_{i=1}^{n} Y_i^{\alpha}$$

$$\lambda^{\alpha} \sum Y_i^{\alpha} = d$$

$$-\frac{\partial^2 L}{\partial \lambda^2}\Big|_{\lambda=\lambda^*} = \frac{\alpha d}{\lambda^{*2}} + \alpha(\alpha-1)\,\frac{d}{\lambda^{*2}}$$

$$\frac{\alpha^2 d}{\lambda^{*2}} \qquad \frac{9 \cdot 5}{0.0018^2} \qquad \underline{13{,}890.00}$$

$$\underline{\underline{Var(\lambda^*)}} \approx \frac{0.0018^2}{45} = \underline{7.2 \cdot 10^{-8}}$$

$$= \underline{0.00027^2} \approx 0.0003^2$$

$$\lambda^* \sim N(\lambda, Var(\lambda^*)) \sim N(\lambda, 0.0003^2)$$

$$\frac{\lambda^* - \lambda}{0.0003} \sim N$$

$$(\lambda^* \pm 1.645 \cdot 0$$

c)

$$P(T > 600) \approx e^{-(0.0018 \cdot 600)^3}$$

$$e^{-1.26} = \underline{\underline{0.284}}$$

Fra Opps 1:

$$R(600) \approx \underline{0.500}$$

e) $H_0 : \lambda = 0.002$

$$L(\alpha, \lambda) = d\left[\ln \alpha + \alpha \ln \lambda + (\alpha - 1) \sum \delta_i \ln(Y_i)\right] - \lambda^{\alpha} \sum_{i=1}^{n} Y_i^{\alpha}$$

$$W = 2\left[L(3, \lambda^*) - L(3, 0.002)\right] \quad \sim \chi_1^2 \text{ under } H_0$$

$$2\left\{d\alpha\left[\ln(0.0018) - \ln(0.002)\right] - \underbrace{\sum_1^n Y_i^{\alpha}}\left[0.0018^3 - 0.002^3\right]\right\}$$

$$\sum_1^n Y_i^{\alpha} = \frac{d}{(\lambda^*)^{\alpha}} \qquad d = 5, \quad \alpha = 3$$

$$\underline{\underline{W}} = 2\left\{5 \cdot 3 \cdot \ln\left(\frac{18}{20}\right) - \frac{5}{0.0018^3}\left[0.0018^3 - 0.002^3\right]\right\}$$

$$= 2\left[-1.58 + \underbrace{5\left[-1 + \left(\frac{20}{18}\right)^3\right]}_{1.85}\right]$$

$= \underline{\underline{0.56}}$ Ikke forkast H_0.

Oppg 3

Overlevelsesfunksjon

A & P s. 66
H & R s. 428, 423

a) (1) $R(t/\underline{s}) = R_0(\lambda(\underline{s})\cdot t)$

der f.eks. $\lambda(s) = \begin{cases} C\cdot s^{a} \\ C e^{-b/s} \\ C s e^{-b/s} \end{cases}$

$$E(T/\underline{s}) = \int_0^\infty R(t/\underline{s})\,dt = \int_0^\infty R_0(\lambda(\underline{s})\cdot t)\,dt$$

$$= \int_0^\infty R_0(u)\frac{du}{\lambda(s)} = \frac{E(T/\underline{s}=\underline{0})}{\lambda(\underline{s})}$$

b)

$$\lambda(\underline{s}) = e^{\beta_1 s_1 + \cdots + \beta_n s_n}$$

Weibull regresjon:

(2) $R(t/\underline{s}) = e^{-[\lambda(\underline{s},\beta)\cdot t]^{\alpha}}$

der $\lambda(\underline{s},\beta) = e^{\beta_1 s_1 + \cdots + \beta_n s_n}$

Altså, skalaparameter, λ, avhenger av stressorer $s_1, \dots, s_n$

(2) spesialtilfelle av (1).

Oppg 4

a posteriori $f(\lambda/x) \propto \frac{(\lambda t)^x}{x!} e^{-\lambda t} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$

$\propto \lambda^{x+\alpha-1} \quad e^{-(t+\beta)\lambda}$ $\sim$ Gamma$(x+\alpha, t+\beta)$

b) $\hat{\lambda} = \frac{X+\alpha}{t+\beta}$ H&R s. 446

a priori $\frac{\alpha}{\beta} = 0.1$

$\beta = 20$ år $\quad \alpha = 2$

c) $\hat{\lambda} = \frac{12+2}{134+20} = \frac{14}{154} = 0.091$

klassisk: $\frac{12}{134} = 0.09$ H&R s 446

$P\left[z_{1-\varepsilon/2,\,2(\alpha+x)} < 2(\beta+t)\lambda < z_{\varepsilon/2,\,2(\alpha+x)}\right] = 1-\varepsilon$

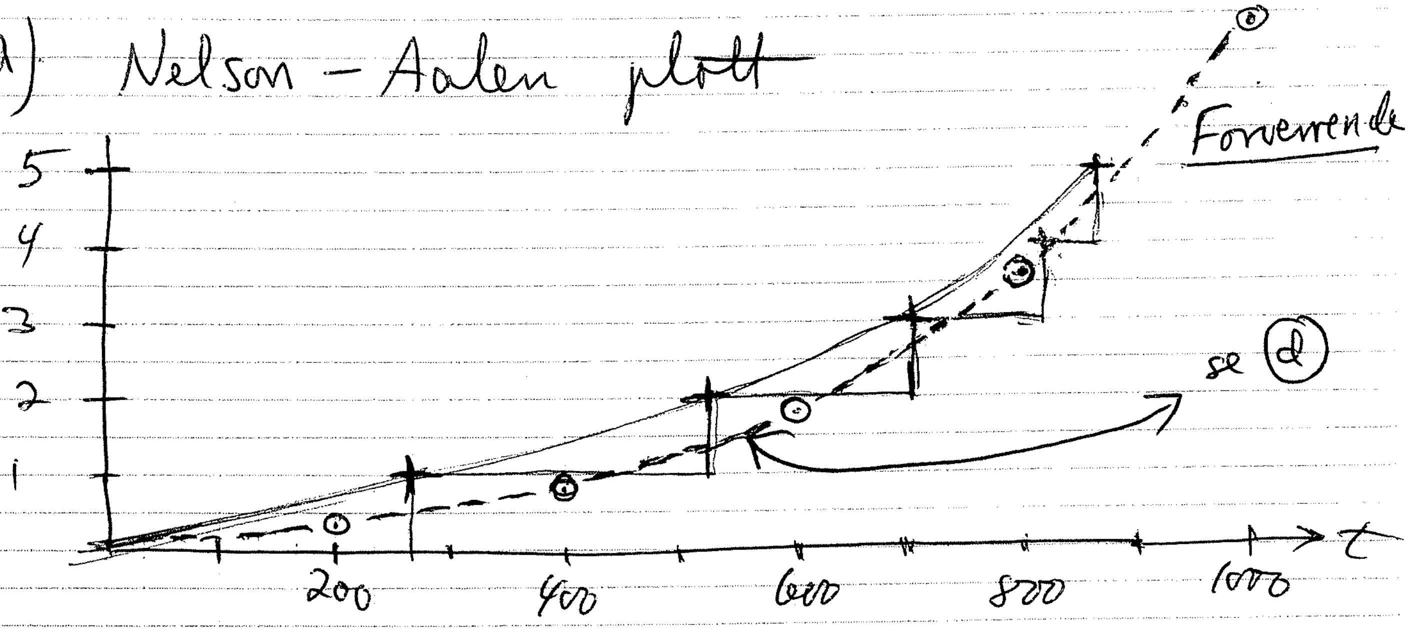
$P\left(\frac{z_{1-\varepsilon/2,\,2(\alpha+x)}}{2(\beta+t)} < \lambda < \frac{z_{\varepsilon/2,\,2(\alpha+x)}}{2(\beta+t)}\right) = 1-\varepsilon$

$\alpha = 2$
$x = 12$
$\beta = 20$
$t = 134$

$\left(\frac{z_{0.95;28}}{2 \cdot 154} < \lambda < \frac{z_{0.05;28}}{2 \cdot 154}\right)$

Oppg 5

a) Nelson - Aalen ~~plott~~



$$\bar{S} = \frac{1}{n} \sum_{1}^{n} S_j \sim N\left(\tau/2, \left(\tau/\sqrt{12n}\right)^2\right)$$

$$U = \frac{\bar{S} - \tau/2}{\tau/\sqrt{12n}} \sim N(0,1)$$

$U > 0$ forverrende
$U < 0$ forbedrende

Får $\underline{U} = \frac{632 - 450}{900/\sqrt{60}} = \underline{1{,}57}$

H_0: ingen trend
Blir ikke forkastet ved nivå 5 %

b) log-linear modell $\underline{\omega(t) = e^{\alpha+\beta t}}$

$$\underline{W(t)} = \frac{1}{\beta}\left[e^{\alpha+\beta t} - e^{\alpha}\right] = \underline{\frac{e^{\alpha}}{\beta}\left[e^{\beta t}-1\right]}$$

$$\beta = 0, \quad \omega(t) = e^{\alpha} \qquad \underline{W(t) = e^{\alpha}\cdot t}$$

c) $$\ell = \omega(s_1)\cdots\omega(s_n)\, e^{-\int_0^{\tau}\omega(t)\,dt}$$

$$= e^{n\alpha+\beta\sum_1^n s_i}\cdot e^{-e^{\alpha}\left[e^{\beta\tau}-1\right]/\beta}$$

$$L = \ln(\ell) = n\alpha + \beta\cdot\sum_1^n s_i - e^{\alpha}\left[e^{\beta\tau}-1\right]/\beta$$

(1) $$\frac{\partial(L)}{\partial\beta} = \sum_1^n s_i - e^{\alpha}\left\{\frac{1}{\beta}\cdot\tau e^{\beta\tau} - \frac{1}{\beta^2}\left(e^{\beta\tau}-1\right)\right\}$$

(2) $$\frac{\partial L}{\partial\alpha} = n - e^{\alpha}\left[e^{\beta\tau}-1\right]/\beta$$

$$(2) \to (2')\quad \underbrace{e^{\alpha^*} = \frac{n\beta^*}{e^{\beta^*\tau}-1}}$$ innsatt i (1)

$$0 = \sum_1^n s_i - \frac{n\beta^*}{e^{\beta^*\tau}-1}\left\{\frac{1}{\beta^*}\tau e^{\beta^*\tau} - \frac{1}{\beta^{*2}}\left(e^{\beta^*\tau}-1\right)\right\}$$

$$0 = \sum_1^n s_i - \frac{n\tau e^{\beta^*\tau}}{e^{\beta^*\tau}-1} + \frac{n}{\beta^*} \qquad \underline{n = N(\tau)}$$

(1') $$\underbrace{\sum_1 s_i + \frac{N(\tau)}{\beta^*} = \frac{N(\tau)\cdot\tau}{1-e^{-\beta^*\tau}}}$$ for β^*

$$(2') \quad \alpha^* = \ln\left[\frac{N(\tau)\,\beta^*}{e^{\beta^*\tau}-1}\right]$$

d) Sett $\Sigma S_i = 3160$ $n = N(\tau) = 5$ $\tau = 900$

Av (1') finnes $\beta^* \approx 0.00301 \approx 0.003$
(numerisk)

$$\alpha^* = \ln\left[\frac{5 \cdot 0.003}{e^{0.003 \cdot 900}-1}\right] \approx -6.83$$

$$\hat{W}(t) = \frac{e^{-6.83}}{0.003}\left[e^{0.003 \cdot t}-1\right] = 0.36\left[e^{0.003 \cdot t}-1\right]$$

	$\hat{W}(t)$
$t = 200$	0.3
$t = 400$	0.84
$t = 600$	1.8
$t = 800$	3.6
$t = 1000$	6.9

Bra overensstemmelse