Mai 2001 LEVETIDSANALYSE

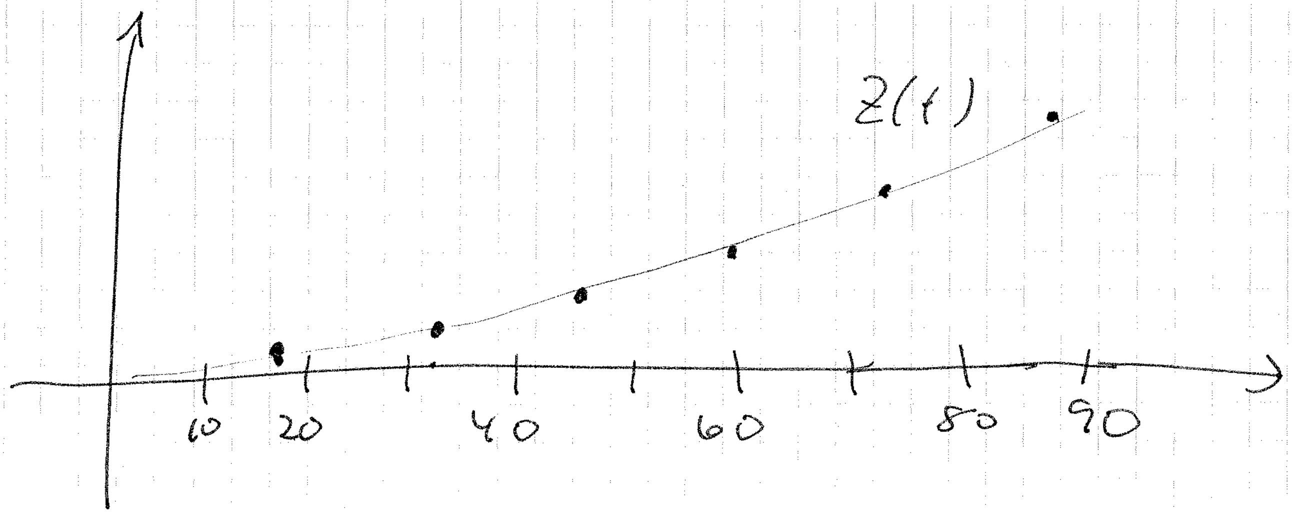
Oppg 1.

a) Sens. modell : "Type $\underline{IV}$" — Jawh-sensurij.

Nelson's estimator

Lag tabellen

	Under risk	Bidrag	$\hat{Z}(t)$
18	8	1/8	1/8
32	7	1/7	1/8 + 1/7
45	6	1/6	$1/8 + 1/7 + \frac{1}{6}$
55*	5		
60	4	1/4	$\frac{1}{8}+\frac{1}{7}+\frac{1}{6}+\frac{1}{4}$
73	3	1/3	$\frac{1}{8}+\frac{1}{7}+\frac{1}{6}+\frac{1}{4}+\frac{1}{3}$
87	2	1/2	$\frac{1}{8}+\frac{1}{7}+\frac{1}{6}+\frac{1}{4}+\frac{1}{3}+\frac{1}{2}$
90*	1		



b) $z(t) = \dfrac{f(t)}{R(t)} = \dfrac{-R'(t)}{R(t)}$

$$\Rightarrow Z(t) = -\int_0^t \frac{R'(u)}{R(u)}\,du$$

Sett $y = R(u)$

$dy = R'(u)\,du$

$$= -\int_{R(0)}^{R(t)} \frac{dy}{y} = -\Big|_{R(0)}^{R(t)} \ln(y)$$

$$= -\ln R(t) + \ln \underbrace{R(0)}_{1}$$

$$= -\ln R(t)$$

$$\Rightarrow e^{-Z(t)} = R(t)$$

$$\Rightarrow \hat{R}_{Nelson}(t) = e^{-\hat{Z}_{Nels}(t)}$$

Oppgave 2

$T_1, \dots, T_n$

a) $f(t;\lambda) = \frac{1}{2}\lambda^3 t^2 e^{-\lambda t}$

Dette er Gamma$(3, \lambda)$

(Gen Gamma(a,b): $\dfrac{b^a t^{a-1} e^{-bt}}{\Gamma(a)}$

Setter her $a=3$, $b=\lambda$

$\Gamma(3) = 2! = 2$

Vet da at T er sum av ~~tre~~ uavh eksp(λ)

$\Rightarrow E(T) = \dfrac{3}{\lambda}$

b) $\hat{}$ Likelihood:

$$L(\lambda) = \prod_{i=1}^{n} \frac{1}{2}\lambda^3 t_i^2 e^{-\lambda t_i}$$

$$= \frac{1}{2^n}\cdot\lambda^{3n}\cdot\left(\prod_{i=1}^{n} t_i\right)^2\cdot e^{-\lambda\sum t_i}$$

$$l(\lambda) = -n\ln 2 + 3n\ln\lambda + 2\sum_{i=1}^{n}\ln t_i - \lambda\sum t_i$$

$$\frac{\partial \ell(\lambda)}{\partial \lambda} = \frac{3n}{\lambda} - \sum t_i = 0$$

$$\Rightarrow \hat{\lambda} = \cancel{\sum t_i}\ \frac{3n}{\sum t_i} = \frac{3}{\bar{t}} = \underline{0.8824}$$

$$\frac{\partial^2 \ln(\lambda)}{\partial \lambda^2} = -\frac{3n}{\lambda^2}$$

$$\Rightarrow \mathrm{Var}(\hat{\lambda}) \approx \left(-\frac{\partial^2 \ln(\lambda)}{\partial \lambda^2}\right)^{-1} = \frac{\lambda^2}{3n}$$

da $\hat{\lambda} \approx N\left(\lambda, \frac{\lambda^2}{3n}\right)$

90% konf int: $\hat{\lambda} \pm 1.645 \cdot \frac{\hat{\lambda}}{\sqrt{3n}}$ Fink

Ellers:

$$\ln\hat{\lambda} \approx N\left(\ln\lambda, (\ln\lambda')^2 \cdot \frac{\lambda^2}{3n}\right)$$

$$= N\left(\ln\lambda, \frac{1}{3n}\right)$$

etc etc.

d) $H_0: \lambda = 1.0$ mot $\lambda \neq 1.0$.

$$W = 2\left[\ell(\hat{\lambda}) - \ell(1)\right]$$

$$= 2\Big[\cancel{-n\ln 2} + 3n\ln\hat{\lambda} \cancel{+2\sum \ln t_i} - \hat{\lambda}\sum t_i$$
$$\cancel{+n\ln 2 - 3n} \cancel{-2\sum \ln t_i} + \sum t_i\Big]$$

$$= 2\Big[3n\ln\hat{\lambda} - (\hat{\lambda}-1)\underbrace{\sum t_i}_{=\frac{3n}{\hat{\lambda}}}\Big]$$

$$= 2\left[3n\ln\hat{\lambda} - \frac{\hat{\lambda}-1}{\hat{\lambda}}\cdot 3n\right]$$

$$= 2\left[3n\ln\hat{\lambda} - 3n + \frac{3n}{\hat{\lambda}}\right]$$

$$= 6n\left[\ln\hat{\lambda} - 1 + \frac{1}{\hat{\lambda}}\right] \quad \text{Regn ut!}$$

Oppgave 3.

Invers Gamma:

$$f(x) = \frac{b^a}{\Gamma(a)} \left(\frac{1}{x}\right)^{a+1} e^{-\frac{b}{x}} \quad \text{for } \begin{matrix} a>0 \\ b>0 \end{matrix}$$

$$E(X) = \frac{b}{a-1} \quad ; a>1$$

a) $f(t;\theta) = \frac{1}{\theta} e^{-\frac{t}{\theta}}$

$\underline{E(T) = \theta}$

Type II - sensurering:

$$L(\lambda) = \frac{1}{\theta^r} e^{-\frac{\sum t_i}{\theta}}$$

Aposteriori: $\propto \left(\frac{1}{\theta}\right)^{r+a+1} \cdot e^{-\frac{\sum t_i + b}{\theta}}$

dvs $\text{InvG}(r+a, \sum t_i + b)$.

b) • Bayes est:

$$\frac{b}{a-1} \quad : \quad \frac{\sum t_i + b}{r + a - 1}$$

apriori
$\theta \approx$ 5

$\cancel{b} = a = 0$ gir

$$\mathrm{InvG}(r, \underset{10}{\sum t_i})$$

då $b = 10$, $\frac{b}{a-1} = 5$

$a - 1 = 2$

$a = 3$

(c) Vis at hvis $Z = \frac{2b}{X} \sim \chi^2_{2a}$.

$X \sim \mathrm{InG}(a, b)$

sier

Finn credibility interval for θ.

a) $W(t) = E(N(t))$

ROCOF $w(t) \underset{def}{=} W'(t)$

$w(t) = \lim_{h \to 0} \frac{\text{forv. ant. feil i } (t, t+h)}{h}$

$= \lim_{h \to 0} \frac{P(\text{feil i } (t, t+h))}{h}$

<u>NHPP:</u> $N(t) \sim \text{Poisson}(W(t))$.

b) Nelson-Aalen.
Laplace-test.

<u>Til grunn:</u> Teste H_0: HPP mot H_1: <u>Ikke</u>.