Mai 2001, 3c   Solution

$$Z = \frac{2b}{X}$$

$$z = \frac{2b}{x} = g(x) \Rightarrow x = \frac{2b}{z} = h(z)$$

Density of Z is therefore

$$f_Z(z) = f_X\left(\frac{2b}{z}\right) \cdot \left| h'(z) \right|$$

$$= \frac{b^a}{\Gamma(a)} \left(\frac{z}{2b}\right)^{a+1} e^{-\frac{z}{2}} \cdot \frac{2b}{z^2}$$

$$= \frac{1}{\Gamma(a) \cdot 2^a} z^{a-1} e^{-\frac{z}{2}}$$

which is the density of χ^2_{2a}.

Credibility interval: $\boxed{\text{Need prior distn for } \theta \text{ in months. } \frac{b}{a-1} = 60 \; ; \; b = 120 \Rightarrow a = 3}$

Posterior distn. is $\text{Inv-Gamma}(\overset{r}{6} + \overset{a}{3}, \overset{\Sigma t_i}{460} + \overset{b}{120})$

$\text{Inv-G}(9, 580)$

Thus

$$P\left(\chi^2_{18,0.95} < \frac{2 \cdot 580}{\Theta} < \chi^2_{18,0.05}\right) = 0.90$$

$$P\left(9.39 < \frac{2 \cdot 580}{\Theta} < 28.87\right) = 0.90$$

$$P\left(\frac{2 \cdot 580}{28.87} < \Theta < \frac{2 \cdot 580}{9.39}\right) = 0.90$$

$$P\left(\cancel{32.58}\ 40.18 < \Theta < \cancel{100.12}\ 123.54\right) = 0.90$$

CREDIB. INTERVAL!