Estimation of σ_{effect} by Lenth's method: The Pseudo Standard Error

Let $C_1, C_2, \ldots, C_m$ be estimated effects, e.g. $\hat{A}, \hat{B}, \widehat{AB}$, etc.

1. Order absolute values $|C_j|$ in increasing order.
2. Find the median of the $|C_j|$ and compute preliminary estimate
$$s_0 = 1.5 \cdot \text{median}_j |C_j|$$
3. Take out the effects C_j with $|C_j| \geq 2.5 \cdot s_0$ and find the median of the rest of the $|C_j|$. Then PSE is this median multiplied by 1.5, i.e.
$$\text{PSE} = 1.5 \cdot \text{median}\{|C_j| : |C_j| < 2.5 s_0\}$$
and this is Lenth's estimate of σ_{effect}.
4. Lenth has suggested empirically that the degrees of freedom to be used with PSE is $m/3$ where m is the initial number of effects in the algorithm. Thus we claim as significant the effects for which $|C_j| > t_{\alpha/2,m/3} \cdot PSE$.

Example: 2^3 experiment in Note

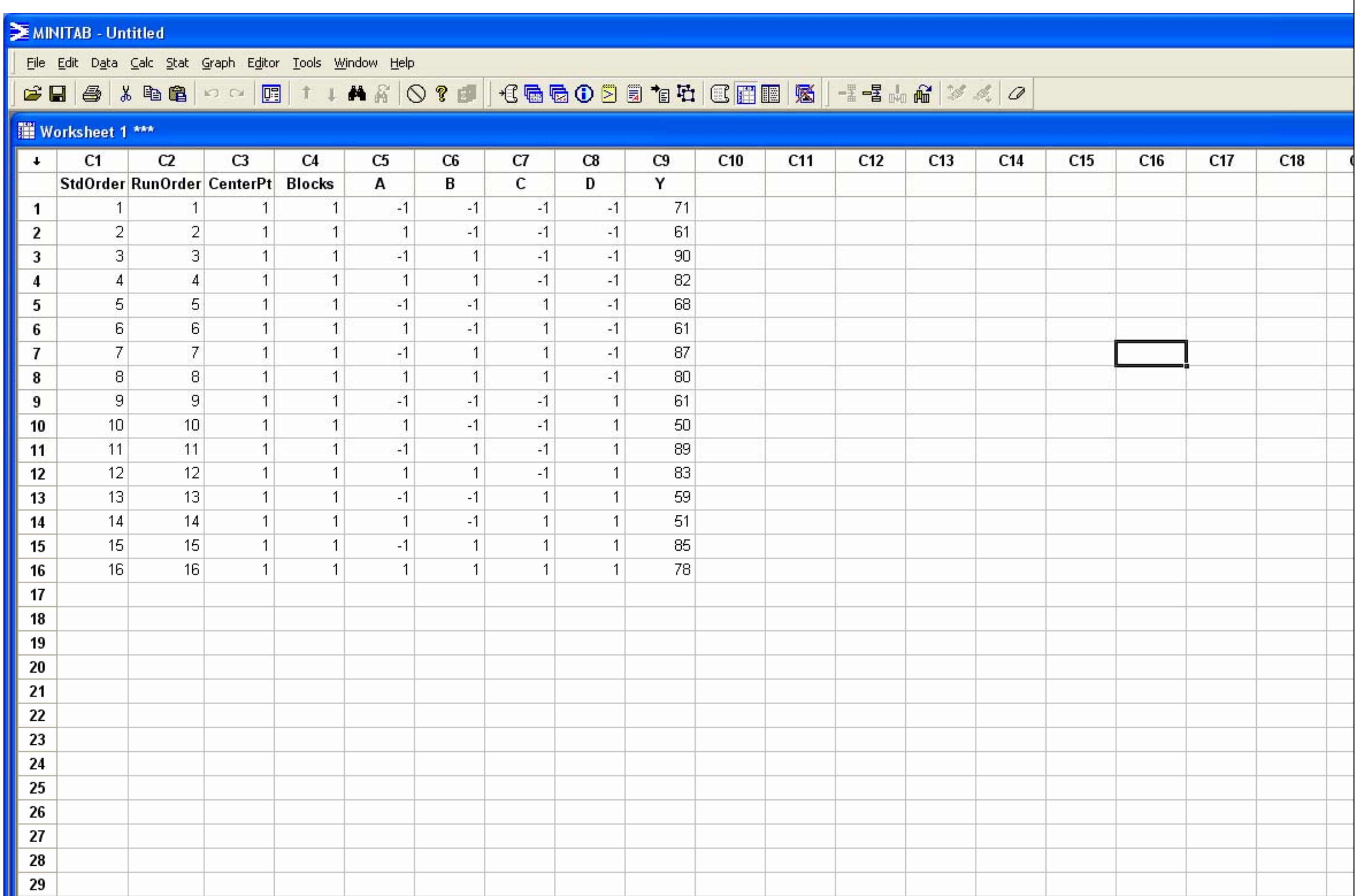
There are $m = 7$ estimated effects.

1. Ordered estimated absolute effects:
$$0, 0.5, 1.5, 1.5, 5, 10, 23$$
2. Median is 1.5 so $s_0 = 1.5 \cdot 1.5 = 2.25$.
3. Throw out large effects, i.e. the ones that are
$$\geq 2.5 \cdot 2.25 = 5.625$$
leaving us with $0, 0.5, 1.5, 1.5, 5$ for which median is still 1.5, so
$$\text{PSE} = 1.5 \cdot 1.5 = 2.25$$
4. Lenth's degrees of freedom is $m/3 = 7/3 = 2.33$, so we claim effects to be significant at 5% level when
$$|C_j| > t_{0.025,2.33} \cdot 2.25 = 3.765 \cdot 2.25 = 8.47.$$

Some theoretical considerations

- The basic underlying idea is that many of the true effects are zero, and that (most of) the ones that are not zero are thrown out in the last step of the algorithm.
- The reason for 1.5 is that if $C \sim N(0, \sigma^2_{effect})$ then the median of the distribution of $|C|$ is $0.675\sigma_{effect}$, so that the median of the distribution of $1.5 \cdot |C|$ is
$$1.5 \cdot 0.675\sigma_{effect} \approx \sigma_{effect}.$$

Four factors – example from Notes



	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 C	C8 D	C9 Y
1	1	1	1	1	-1	-1	-1	-1	71
2	2	2	1	1	1	-1	-1	-1	61
3	3	3	1	1	-1	1	-1	-1	90
4	4	4	1	1	1	1	-1	-1	82
5	5	5	1	1	-1	-1	1	-1	68
6	6	6	1	1	1	-1	1	-1	61
7	7	7	1	1	-1	1	1	-1	87
8	8	8	1	1	1	1	1	-1	80
9	9	9	1	1	-1	-1	-1	1	61
10	10	10	1	1	1	-1	-1	1	50
11	11	11	1	1	-1	1	-1	1	89
12	12	12	1	1	1	1	-1	1	83
13	13	13	1	1	-1	-1	1	1	59
14	14	14	1	1	1	-1	1	1	51
15	15	15	1	1	-1	1	1	1	85
16	16	16	1	1	1	1	1	1	78

Full Factorial Design

Factors: 4 Base Design:
4; 16
Runs: 16 Replicates:
1
Blocks: 1 Center pts
(total): 0

All terms are free from
aliasing.

Factorial Fit: Y versus A; B; C; D

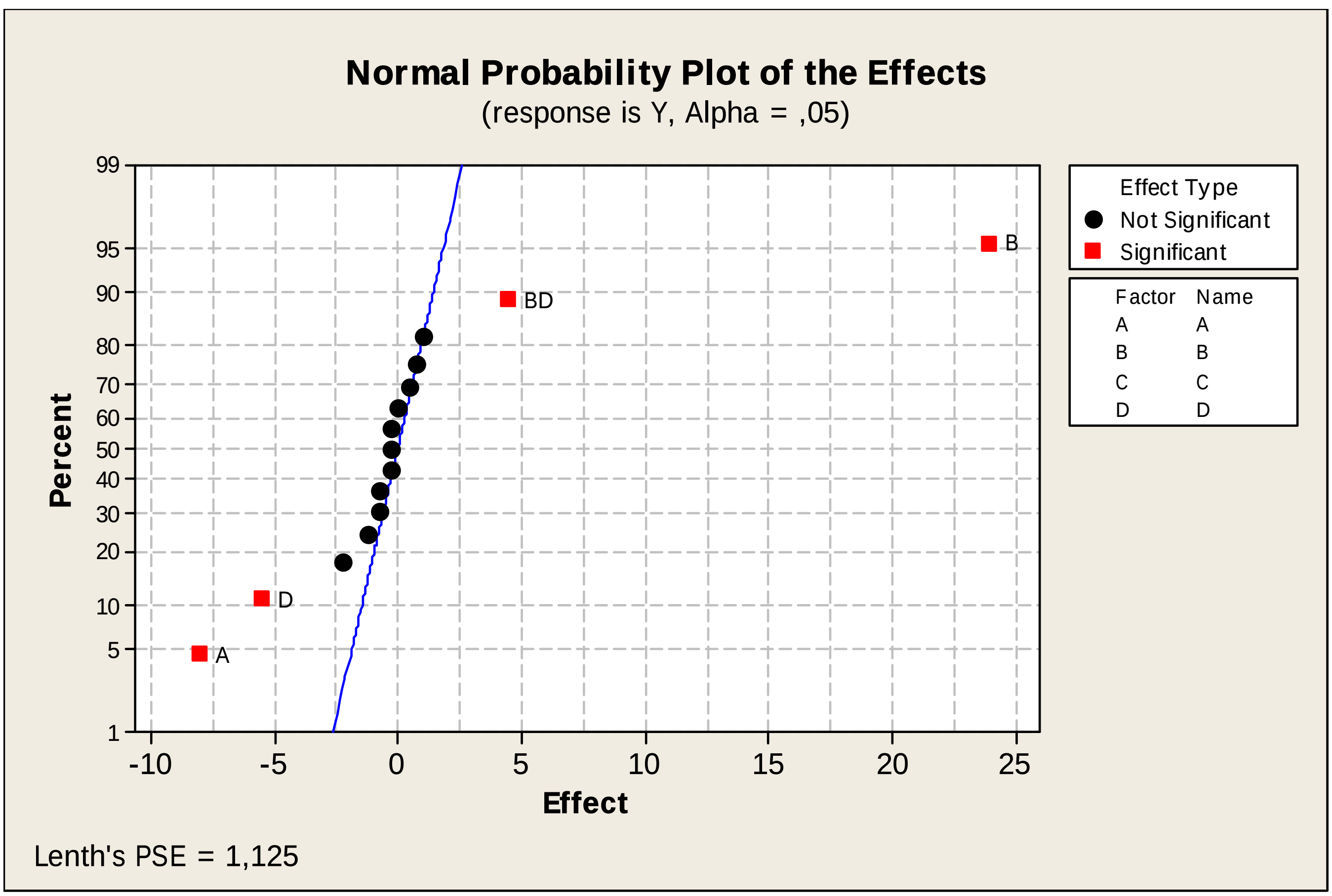
Estimated Effects and Coefficients for Y (coded units)

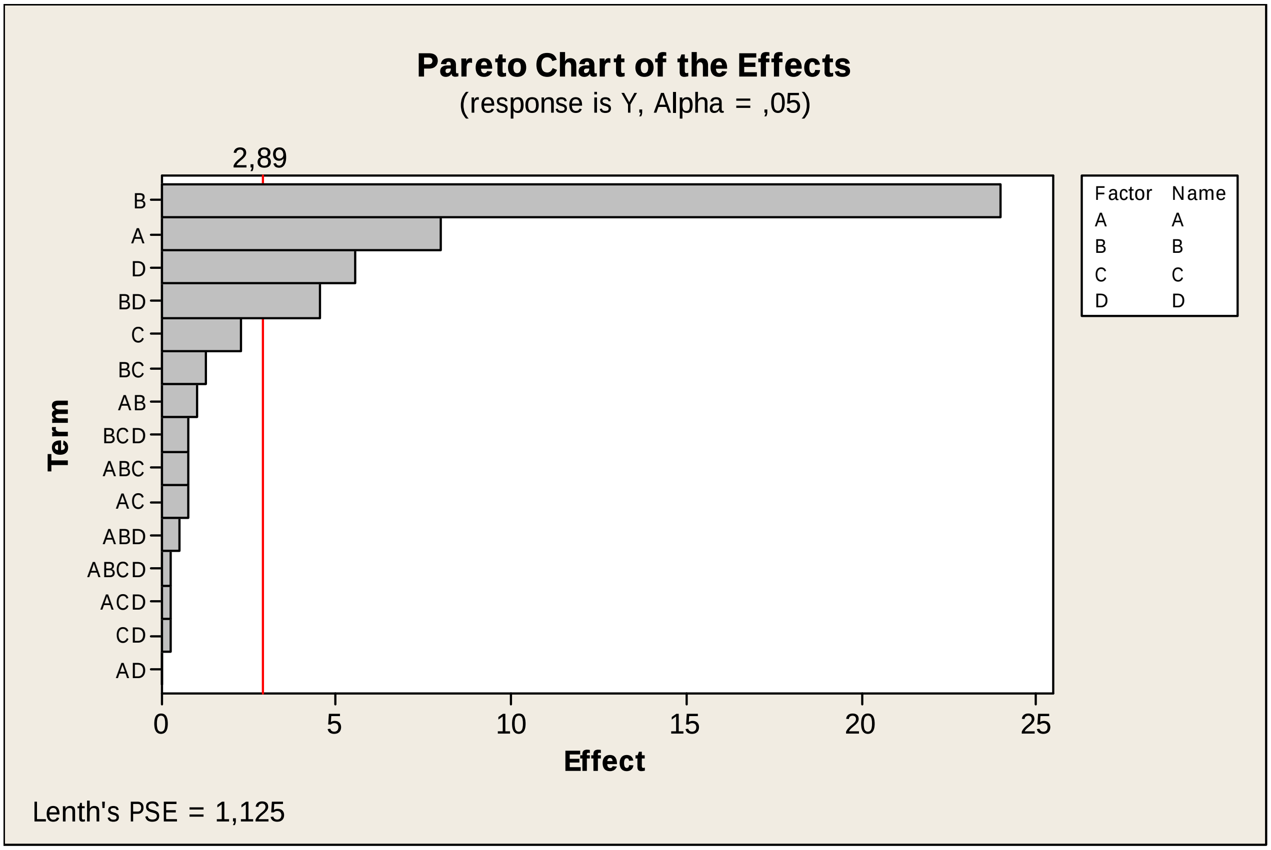
Term	Effect	Coef
Constant		72,250
A	-8,000	-4,000
B	24,000	12,000
C	-2,250	-1,125
D	-5,500	-2,750
A*B	1,000	0,500
A*C	0,750	0,375
A*D	-0,000	-0,000
B*C	-1,250	-0,625
B*D	4,500	2,250
C*D	-0,250	-0,125
A*B*C	-0,750	-0,375
A*B*D	0,500	0,250
A*C*D	-0,250	-0,125
B*C*D	-0,750	-0,375
A*B*C*D	-0,250	-0,125

S = *

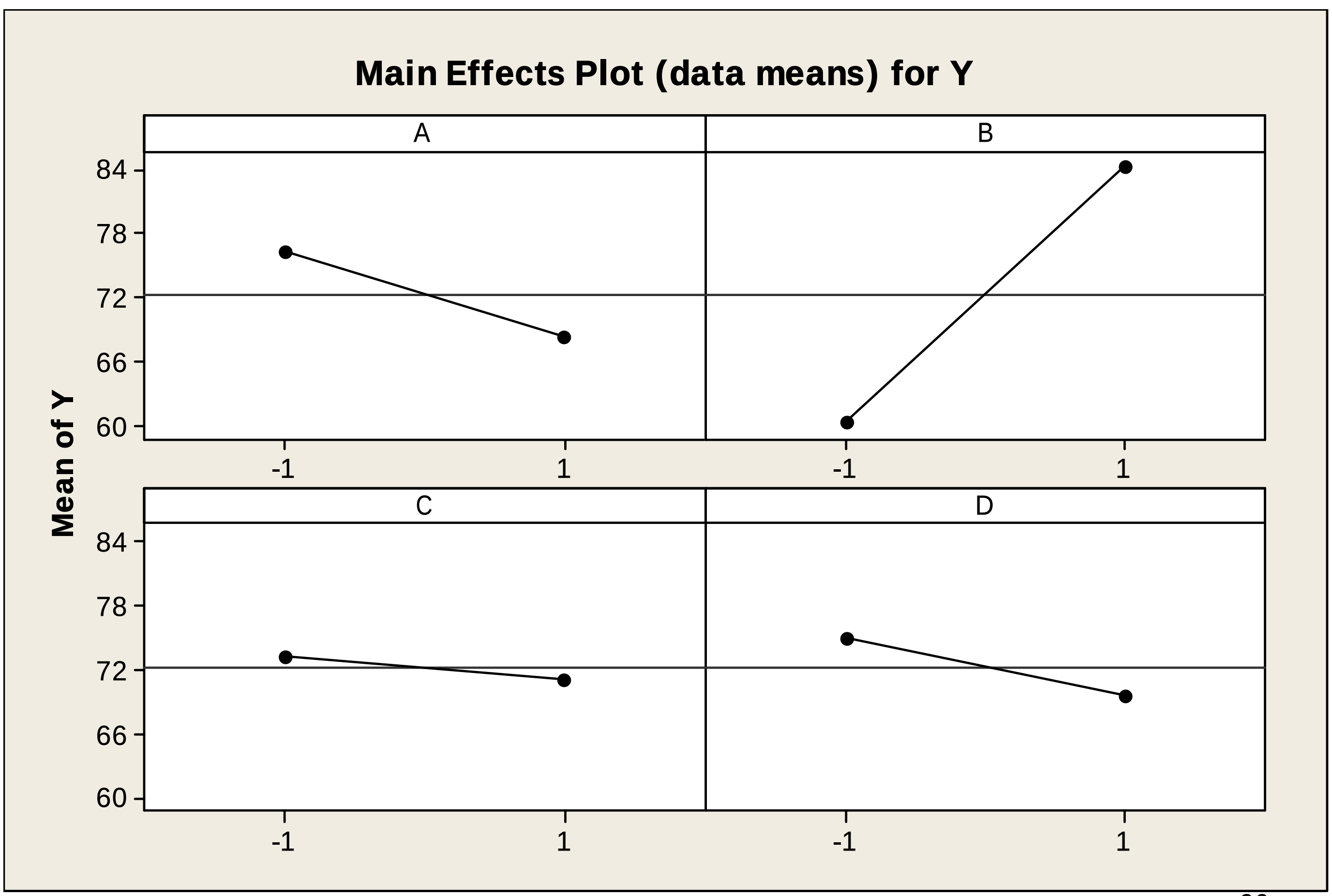
Analysis of Variance for Y (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	4	2701,25	2701,25	675,313	*	*
2-Way Interactions	6	93,75	93,75	15,625	*	*
3-Way Interactions	4	5,75	5,75	1,438	*	*
4-Way Interactions	1	0,25	0,25	0,250	*	*
Residual Error	0	*	*	*		
Total	15	2801,00				

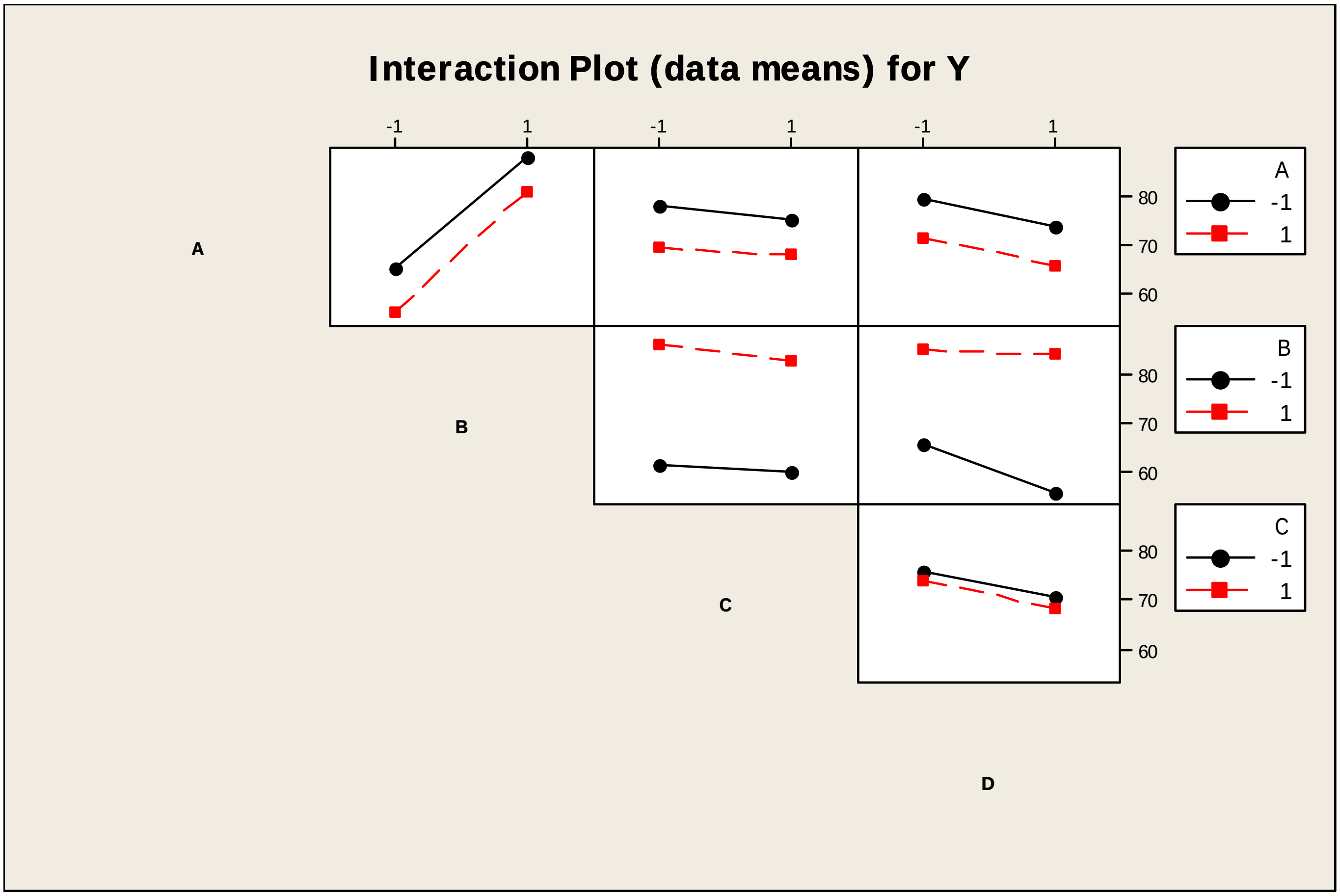




Pareto Chart of the Effects
(response is Y, Alpha = ,05)
2,89
B
A
D
BD
C
BC
AB
BCD
ABC
AC
ABD
ABCD
ACD
CD
AD
Term
0
5
10
15
20
25
Effect
Factor Name
A A
B B
C C
D D
Lenth's PSE = 1,125



Main Effects Plot (data means) for Y
A
B
C
D
Mean of Y
84
78
72
66
60
-1
1



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Example: Three factors and replicate

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Worksheet 1 ***

↓	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 C	C8 Y	C9	C10	C11	C12	C13	C14	C15
1	1	1	1	1	-1	-1	-1	59							
2	2	2	1	1	1	-1	-1	74							
3	3	3	1	1	-1	1	-1	50							
4	4	4	1	1	1	1	-1	69							
5	5	5	1	1	-1	-1	1	50							
6	6	6	1	1	1	-1	1	81							
7	7	7	1	1	-1	1	1	46							
8	8	8	1	1	1	1	1	79							
9	9	9	1	1	-1	-1	-1	61							
10	10	10	1	1	1	-1	-1	70							
11	11	11	1	1	-1	1	-1	58							
12	12	12	1	1	1	1	-1	67							
13	13	13	1	1	-1	-1	1	54							
14	14	14	1	1	1	-1	1	85							
15	15	15	1	1	-1	1	1	44							
16	16	16	1	1	1	1	1	81							
17															
18															
19															
20															

Factorial Fit: Y versus A; B; C

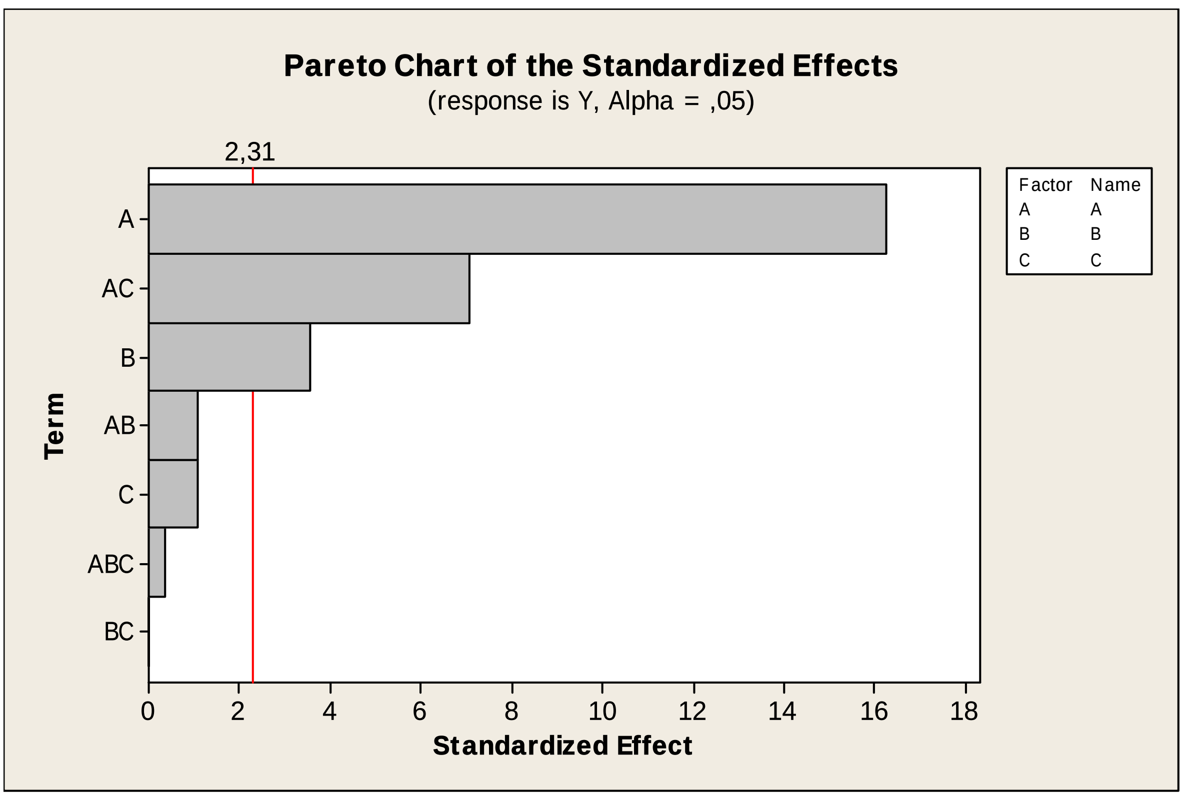
Estimated Effects and Coefficients for Y (coded units)

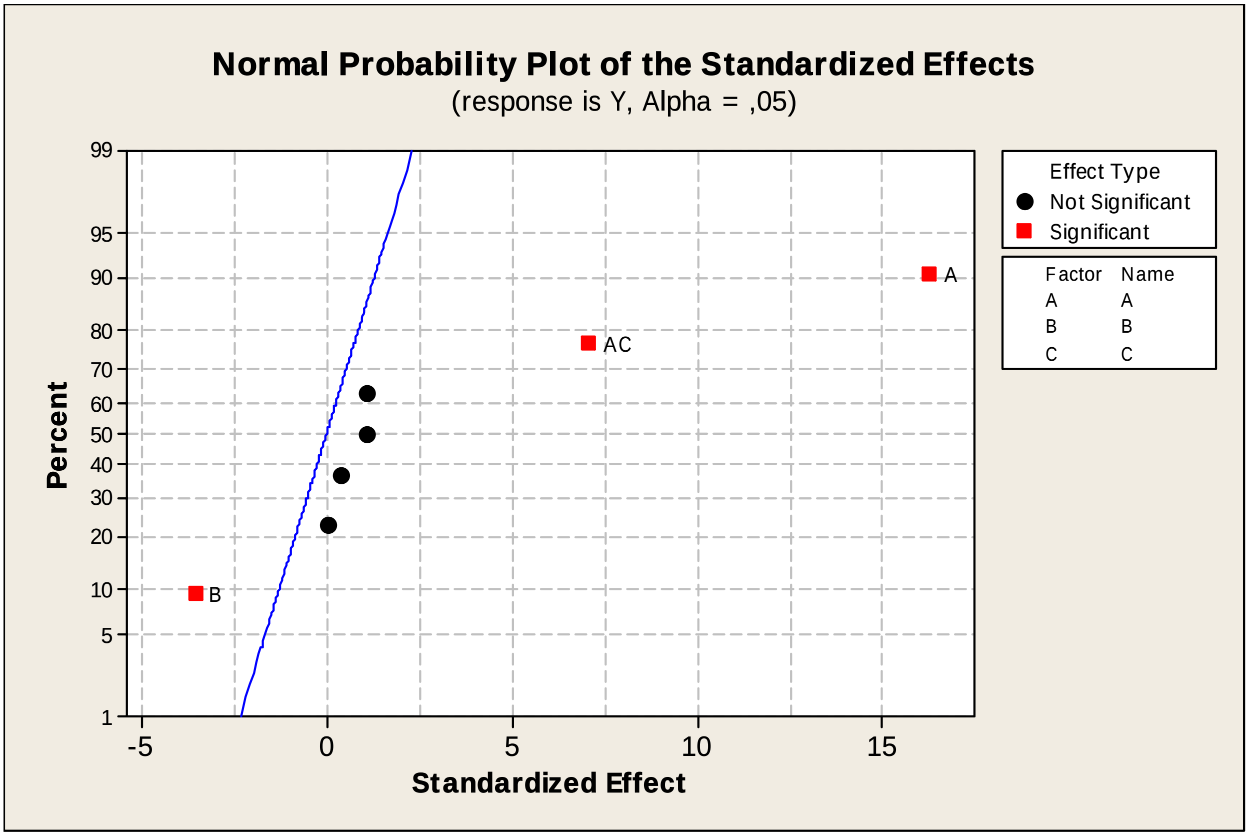
Term	Effect	Coef	SE Coef	T	P
Constant		64,250	0,7071	90,86	0,000
A	23,000	11,500	0,7071	16,26	0,000
B	-5,000	-2,500	0,7071	-3,54	0,008
C	1,500	0,750	0,7071	1,06	0,320
A*B	1,500	0,750	0,7071	1,06	0,320
A*C	10,000	5,000	0,7071	7,07	0,000
B*C	0,000	0,000	0,7071	0,00	1,000
A*B*C	0,500	0,250	0,7071	0,35	0,733

S = 2,82843   R-Sq = 97,63%   R-Sq(adj) = 95,55%

Analysis of Variance for Y (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	2225,00	2225,00	741,667	92,71	0,000
2-Way Interactions	3	409,00	409,00	136,333	17,04	0,001
3-Way Interactions	1	1,00	1,00	1,000	0,13	0,733
Residual Error	8	64,00	64,00	8,000		
Pure Error	8	64,00	64,00	8,000		
Total	15	2699,00				





Eksempel. 2^3 forsøk med gjentak

A	B	C	y_{i1}	y_{i2}	$y_{i1} - y_{i2}$	$\frac{(y_{i2} - y_{i1})^2}{2}$
-	-	-	59	61	-2	2
+	-	-	74	70	4	8
-	+	-	50	58	-8	32
+	+	-	69	67	2	2
-	-	+	50	54	-4	8
+	-	+	81	85	-4	8
-	+	+	46	44	2	2
+	+	+	79	81	-2	2
Totalt						64

Estimatet for σ^2 blir då: $s^2 = \frac{64}{8} = 8$.

$$\sigma^2_{effekt} = \frac{4\sigma^2}{n} \Rightarrow s^2_{effekt} = \frac{4 \cdot s^2}{16} = \frac{4 \cdot 8}{16} = 2$$