**TABLE 14.1** Two-Factor Experiment with $n$ Replications

| $A$ | $B$: | 1 | 2 | $\cdots$ | $b$ | Total | Mean |
|---|---|---|---|---|---|---|---|
| 1 | | $y_{111}$ | $y_{121}$ | $\cdots$ | $y_{1b1}$ | $Y_{1..}$ | $\bar{y}_{1..}$ |
| | | $y_{112}$ | $y_{122}$ | $\cdots$ | $y_{1b2}$ | | |
| | | $\vdots$ | $\vdots$ | | $\vdots$ | | |
| | | $y_{11n}$ | $y_{12n}$ | $\cdots$ | $y_{1bn}$ | | |
| 2 | | $y_{211}$ | $y_{221}$ | $\cdots$ | $y_{2b1}$ | $Y_{2..}$ | $\bar{y}_{2..}$ |
| | | $y_{212}$ | $y_{222}$ | $\cdots$ | $y_{2b2}$ | | |
| | | $\vdots$ | $\vdots$ | | $\vdots$ | | |
| | | $y_{21n}$ | $y_{22n}$ | $\cdots$ | $y_{2bn}$ | | |
| $\vdots$ | | $\vdots$ | $\vdots$ | | $\vdots$ | $\vdots$ | $\vdots$ |
| $a$ | | $y_{a11}$ | $y_{a21}$ | $\cdots$ | $y_{ab1}$ | $Y_{a..}$ | $\bar{y}_{a..}$ |
| | | $y_{a12}$ | $y_{a22}$ | $\cdots$ | $y_{ab2}$ | | |
| | | $\vdots$ | $\vdots$ | | $\vdots$ | | |
| | | $y_{a1n}$ | $y_{a2n}$ | $\cdots$ | $y_{abn}$ | | |
| Total | | $Y_{.1.}$ | $Y_{.2.}$ | $\cdots$ | $Y_{.b.}$ | $Y_{...}$ | |
| Mean | | $\bar{y}_{.1.}$ | $\bar{y}_{.2.}$ | $\cdots$ | $\bar{y}_{.b.}$ | | $\bar{y}_{...}$ |



$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk},$$

on which we impose the restrictions

$$\sum_{i=1}^{a}\alpha_i = 0, \qquad \sum_{j=1}^{b}\beta_j = 0, \qquad \sum_{i=1}^{a}(\alpha\beta)_{ij} = 0, \qquad \sum_{j=1}^{b}(\alpha\beta)_{ij} = 0.$$

The three hypotheses to be tested are as follows:

**1.** $H_0'$: $\alpha_1 = \alpha_2 = \cdots = \alpha_a = 0$,
$H_1'$: At least one of the $\alpha_i$'s is not equal to zero.

**2.** $H_0''$: $\beta_1 = \beta_2 = \cdots = \beta_b = 0$,
$H_1''$: At least one of the $\beta_j$'s is not equal to zero.

**3.** $H_0'''$: $(\alpha\beta)_{11} = (\alpha\beta)_{12} = \cdots = (\alpha\beta)_{ab} = 0$,
$H_1'''$: At least one of the $(\alpha\beta)_{ij}$'s is not equal to zero.

**Theorem 14.1** ***Sum-of-Squares Identity***

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..}-\bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.}-\bar{y}_{...})^2$$
$$+n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.}-\bar{y}_{i..}-\bar{y}_{.j.}+\bar{y}_{...})^2$$
$$+\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk}-\bar{y}_{ij.})^2.$$

Symbolically, we write the sum-of-squares identity as

$$SST = SSA + SSB + SS(AB) + SSE,$$

Degrees of freedom:

$$abn-1 = (a-1)+(b-1)+(a-1)(b-1)+ab(n-1).$$



$$E(S_1^2) = E\left[\frac{SSA}{a-1}\right] = \sigma^2 + \frac{nb\sum_{i=1}^{a}\alpha_i^2}{a-1},$$

$$E(S_2^2) = E\left[\frac{SSB}{b-1}\right] = \sigma^2 + \frac{na\sum_{j=1}^{b}\beta_j^2}{b-1},$$

$$E(S_3^2) = E\left[\frac{SS(AB)}{(a-1)(b-1)}\right] = \sigma^2 + \frac{n\sum_{i=1}^{a}\sum_{j=1}^{b}(\alpha\beta)_{ij}^2}{(a-1)(b-1)},$$

$$E(S^2) = E\left[\frac{SSE}{ab(n-1)}\right] = \sigma^2,$$

***F*-test for factor *A***

$$f_1 = \frac{s_1^2}{s^2}$$

***F*-test for factor *B***

$$f_2 = \frac{s_2^2}{s^2}$$

***F*-test for interaction**

$$f_3 = \frac{s_3^2}{s^2}$$

**TABLE 14.2** Analysis of Variance for the Two-Factor Experiment with $n$ Replications

| Source of variation | Sum of squares | Degrees of freedom | Mean square | Computed $f$ |
|---|---|---|---|---|
| Main effect | | | | |
| $A$ | $SSA$ | $a-1$ | $s_1^2 = \frac{SSA}{a-1}$ | $f_1 = \frac{s_1^2}{s^2}$ |
| $B$ | $SSB$ | $b-1$ | $s_2^2 = \frac{SSB}{b-1}$ | $f_2 = \frac{s_2^2}{s^2}$ |
| Two-factor interactions | | | | |
| $AB$ | $SS(AB)$ | $(a-1)(b-1)$ | $s_3^2 = \frac{SS(AB)}{(a-1)(b-1)}$ | $f_3 = \frac{s_3^2}{s^2}$ |
| Error | $SSE$ | $ab(n-1)$ | $s^2 = \frac{SSE}{ab(n-1)}$ | |
| Total | $SST$ | $abn-1$ | | |



**TABLE 14.3** Propellant Burning Rates

| Missile system | Propellant type: | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|---|
| $a_1$ | | 34.0 | 30.1 | 29.8 | 29.0 |
| | | 32.7 | 32.8 | 26.7 | 28.9 |
| $a_2$ | | 32.0 | 30.2 | 28.7 | 27.6 |
| | | 33.2 | 29.8 | 28.1 | 27.8 |
| $a_3$ | | 28.4 | 27.3 | 29.7 | 28.8 |
| | | 29.3 | 28.9 | 27.3 | 29.1 |

**TABLE 14.4** Analysis of Variance for the Data of Table 14.3

| Source of variation | Sum of squares | Degrees of freedom | Mean square | Computed $f$ |
|---|---|---|---|---|
| Missile system | 14.52 | 2 | 7.26 | 5.84 |
| Propellant type | 40.08 | 3 | 13.36 | 10.75 |
| Interaction | 22.16 | 6 | 3.69 | 2.97 |
| Error | 14.91 | 12 | 1.24 | |
| Total | 91.68 | 23 | | |

**Eksempel 9:**

Man tror at maksimal spenning for en spesiell type lagringsbatterier er avhengig av materialet som benyttes i platene i batteriet og temperaturen i lokalet hvor batteriet er lokalisert. For å undersøke dette ønsker man å gjøre 4 gjentak for hver av tre materialer og tre temperaturer.

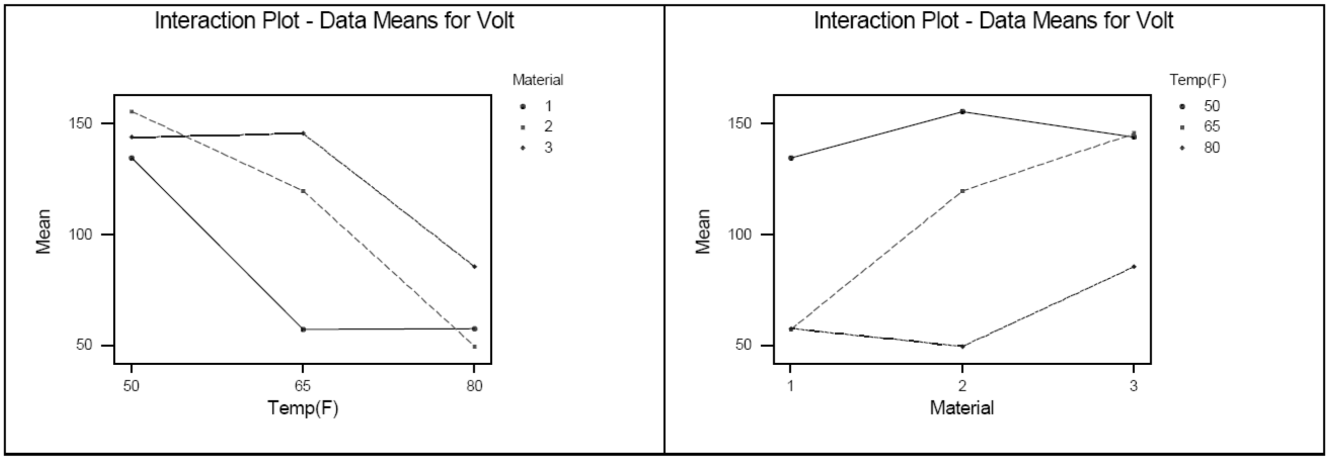
## Tabulated Statistics: Temp(F); Material

```
Rows: Temp(F)     Columns: Material

          1        2        3

50   130.00   150.00   138.00
     155.00   188.00   110.00
      74.00   159.00   168.00
     180.00   126.00   160.00

65    34.00   136.00   174.00
      40.00   122.00   120.00
      80.00   106.00   150.00
      75.00   115.00   139.00

80    20.00    25.00    96.00
      70.00    70.00   104.00
      82.00    58.00    82.00
      58.00    45.00    60.00

 Cell Contents --
            Volt:Data
```

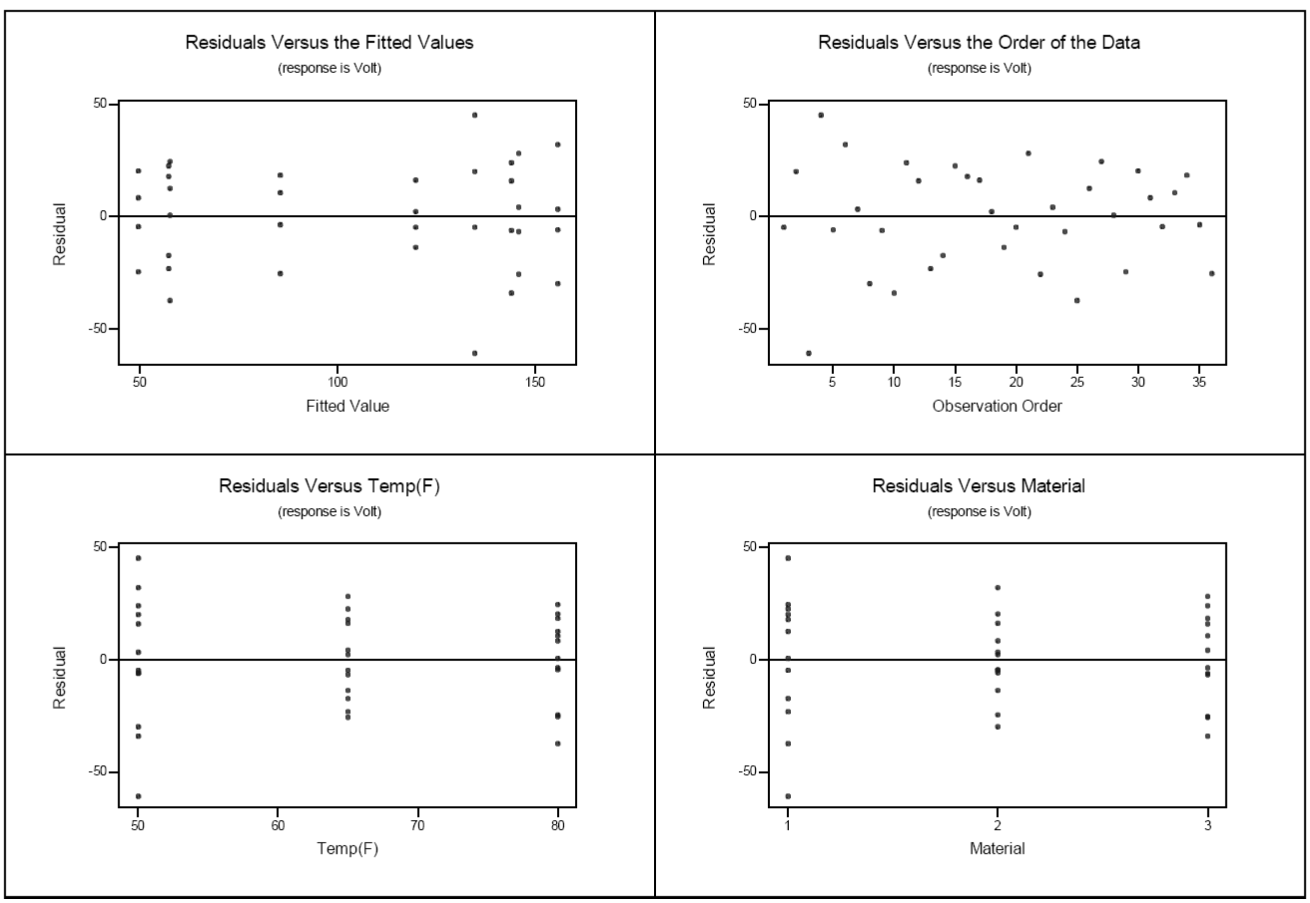


## ANOVA: Volt versus Temp(F); Material

```
Factor      Type Levels Values
Temp(F)    fixed      3     50     65     80
Material   fixed      3      1      2      3


Analysis of Variance for Volt
Source              DF        SS        MS        F       P
Temp(F)              2   39118.7   19559.4    28.97   0.000
Material             2   10683.7    5341.9     7.91   0.002
Temp(F)*Material     4    9613.8    2403.4     3.56   0.019
Error               27   18230.7     675.2
Total               35   77647.0

Means

Temp(F)      N      Volt
50          12    144.83
65          12    107.58
80          12     64.17

Material      N      Volt
1            12     83.17
2            12    108.33
3            12    125.08
```


Interaction Plot - Data Means for Volt
Material
1
2
3
Mean
150
100
50
50
65
80
Temp(F)
Interaction Plot - Data Means for Volt
Temp(F)
50
65
80
Mean
150
100
50
1
2
3
Material

29


Residuals Versus the Fitted Values
(response is Volt)
Residual
50
0
-50
50
100
150
Fitted Value
Residuals Versus the Order of the Data
(response is Volt)
Residual
50
0
-50
5
10
15
20
25
30
35
Observation Order
Residuals Versus Temp(F)
(response is Volt)
Residual
50
0
-50
50
60
70
80
Temp(F)
Residuals Versus Material
(response is Volt)
Residual
50
0
-50
1
2
3
Material

30