13.12 Random Effects Model

Example 13.8 The following data are coded observations on the yield of a chemical process using 5 batches of raw material selected randomly:

Batch:	1	2	3	4	5	
	9.7	10.4	15.9	8.6	9.7	
	5.6	9.6	14.4	11.1	12.8	
	8.4	7.3	8.3	10.7	8.7	
	7.9	6.8	12.8	7.6	13.4	
	8.2	8.8	7.9	6.4	8.3	
	7.7	9.2	11.6	5.9	11.7	
	8.1	7.6	9.8	8.1	10.7	
Total	55.6	59.7	80.7	58.4	75.3	329.7

Model and Assumptions for Random Effects Model

The one-way **random effects model,** often referred to as **model II,** is written like the fixed effects model but with the terms taking on different meanings. The response

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$$

is now a value of the random variable

$$Y_{ij} = \mu + A_i + E_{ij}$$

with $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n$ where the A_i's are normally and independently distributed with mean zero and variance σ_α^2 and are independent of the E_{ij}'s. As for the fixed effects model, the E_{ij}'s are also normally and independently distributed with mean zero and variance σ^2. Note that for a model II experiment, the random variable $\sum_{i=1}^{k} A_i$ assumes the value $\sum_{i=1}^{k} \alpha_i$; and the constraint that these α_i's sum to zero no longer applies.

TABLE 13.17 Expected Mean Squares for the One-Factor Experiment

Source of variation	Degrees of freedom	Mean squares	Expected mean squares Model I	Model II
Treatments	$k-1$	s_1^2	$\sigma^2 + \frac{n\sum_{i=1}^{k}\alpha_i^2}{k-1}$	$\sigma^2 + n\sigma_\alpha^2$
Error	$k(n-1)$	σ^2	σ^2	σ^2
Total	$nk-1$			

$$f = \frac{s_1^2}{s^2}$$

Solution The total, batch, and error sum of squares are

$$SST = 194.64, \quad SSA = 72.60, \quad SSE = 194.64 - 72.60 = 122.04.$$

These results, with the remaining computations, are shown in Table 13.18. The f-ratio is significant at the $\alpha = 0.05$ level, indicating that the hypothesis of a zero batch component is rejected. An estimate of the batch variance component is

$$\hat{\sigma}_\alpha^2 = \frac{18.15 - 4.07}{7} = 2.01.$$

TABLE 13.18 Analysis of Variance for Example 13.8

Source of variation	Sum of squares	Degrees of freedom	Mean square	Computed f
Batches	72.60	4	18.15	4.46
Error	122.04	30	4.07	
Total	194.64	34		

Note that while the **batch variance component** is significantly different from zero, when gauged against the estimate of σ^2, namely $\hat{\sigma}^2 = \text{MSE} = 4.07$, it appears as if the batch variance component is not appreciably large.

MINITAB (Example 13.8)

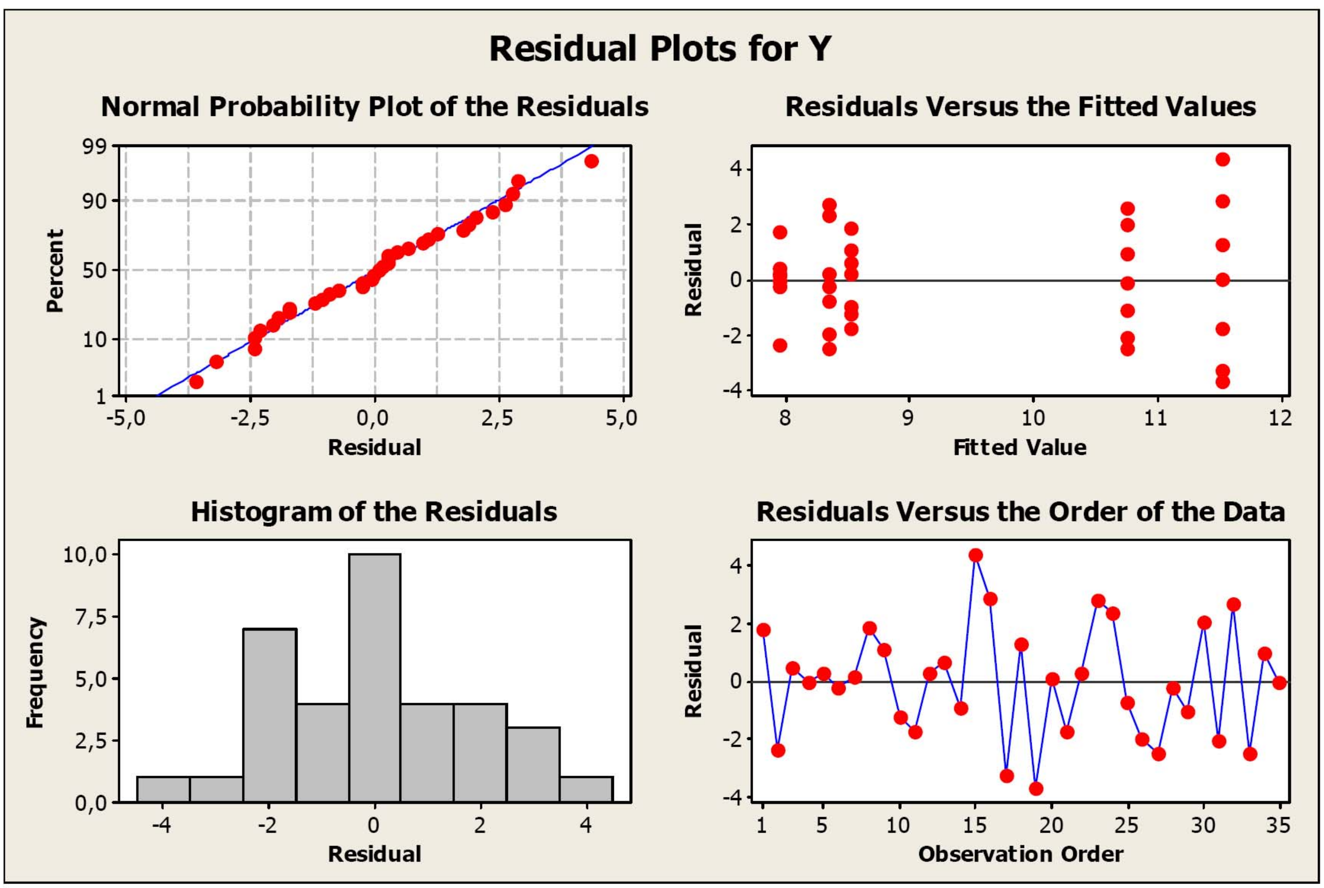
ANOVA: Y versus A

Factor	Type	Levels	Values
A	random	5	1; 2; 3; 4; 5

Analysis of Variance for Y

Source	DF	SS	MS	F	P
A	4	72,596	18,149	4,46	0,006
Error	30	122,040	4,068		
Total	34	194,636			

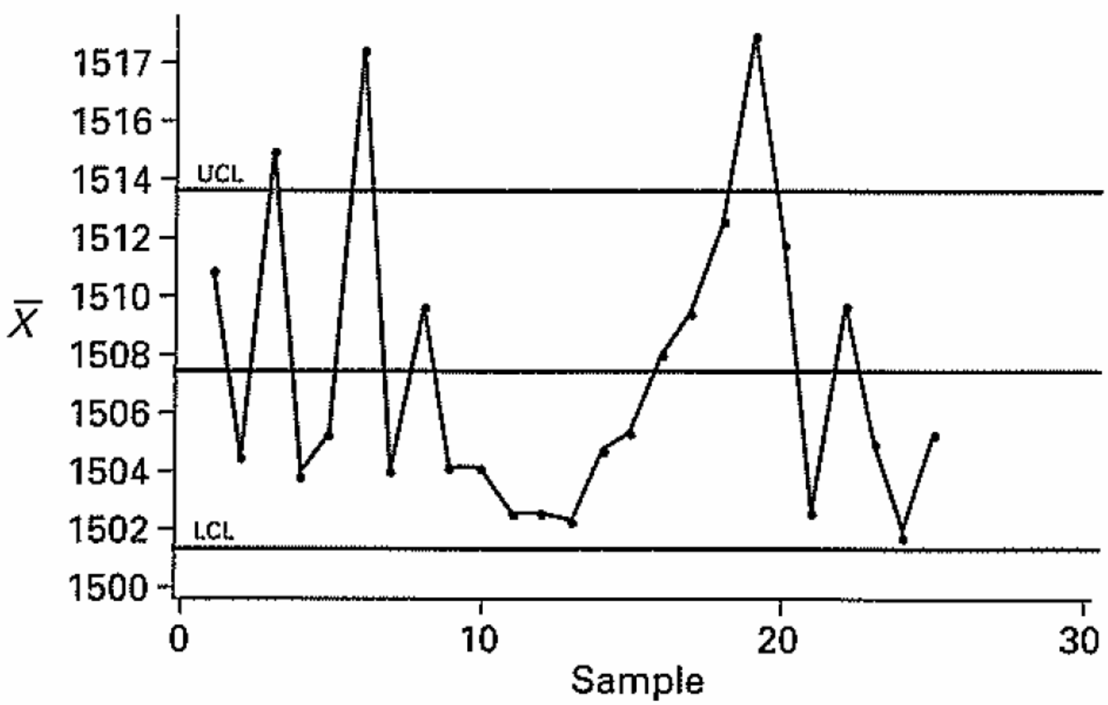
S = 2,01693   R-Sq = 37,30%   R-Sq(adj) = 28,94%



Control Charts

TABLE 17.1 Sample Information on Tensile Strength Data

Sample number	Observations					$\overline{X}_i$	R_i
1	1515	1518	1512	1498	1511	1510.8	20
2	1504	1511	1507	1499	1502	1504.6	12
3	1517	1513	1504	1521	1520	1515.0	17
4	1497	1503	1510	1508	1502	1504.0	13
5	1507	1502	1497	1509	1512	1505.4	15
6	1519	1522	1523	1517	1511	1518.4	12
7	1498	1497	1507	1511	1508	1504.2	14
8	1511	1518	1507	1503	1509	1509.6	15
9	1506	1503	1498	1508	1506	1504.2	10
10	1503	1506	1511	1501	1500	1504.2	11
11	1499	1503	1507	1503	1501	1502.6	8
12	1507	1503	1502	1500	1501	1502.6	7
13	1500	1506	1501	1498	1507	1502.4	9
14	1501	1509	1503	1508	1503	1504.8	8
15	1507	1508	1502	1509	1501	1505.4	8
16	1511	1509	1503	1510	1507	1508.0	8
17	1508	1511	1513	1509	1506	1509.4	7
18	1508	1509	1512	1515	1519	1512.6	11
19	1520	1517	1519	1522	1516	1518.8	6
20	1506	1511	1517	1516	1508	1511.6	11
21	1500	1498	1503	1504	1508	1502.6	10
22	1511	1514	1509	1508	1506	1509.6	8
23	1505	1508	1500	1509	1503	1505.0	9
24	1501	1498	1505	1502	1505	1502.2	7
25	1509	1511	1507	1500	1499	1505.2	12



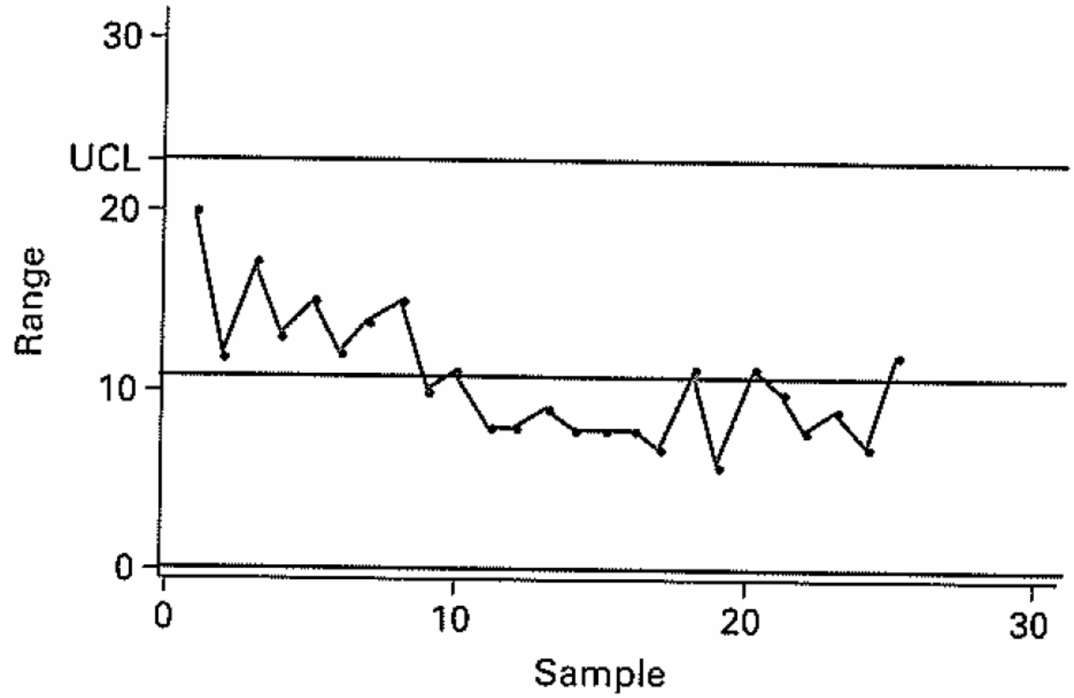


Figure 17.3 *R*-chart for the tensile strength example.

Control Charts for Defects (Poisson)

$$\text{UCL} = \hat{\lambda} + 3\sqrt{\hat{\lambda}} = 13.2678, \qquad \text{LCL} = \hat{\lambda} - 3\sqrt{\hat{\lambda}} = -1.3678 \quad \text{(LCL set to zero)}.$$

TABLE 17.4 Data for Example 17.5; Control Involves Number of Defects in Sheet Metal Rolls

Sample number	Number of defects	Sample number	Number of defects
1	8	11	3
2	7	12	7
3	5	13	5
4	4	14	9
5	4	15	7
6	7	16	7
7	6	17	8
8	4	18	6
9	5	19	7
10	6	20	4
			Ave. 5.95

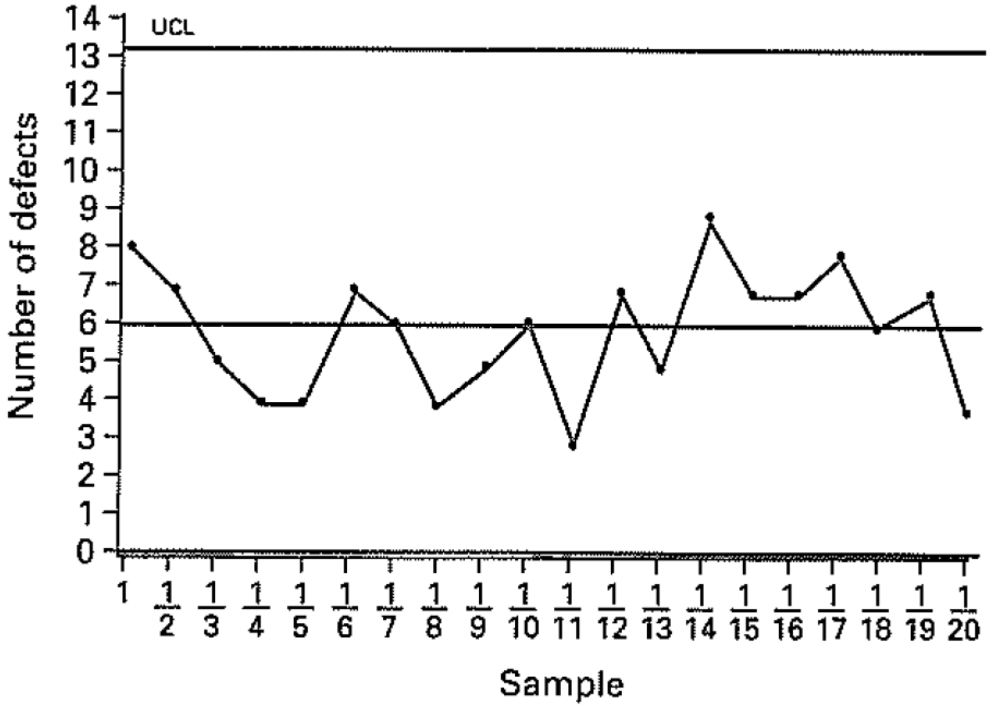


Figure 17.9 Preliminary data plotted on the control chart for Example 17.5.

TABLE 17.5 Additional Data from the Production Process of Example 17.5

Sample number	Number of defects	Sample number	Number of defects
1	3	11	7
2	5	12	5
3	8	13	9
4	5	14	4
5	8	15	6
6	4	16	5
7	3	17	3
8	6	18	2
9	5	19	1
10	2	20	6

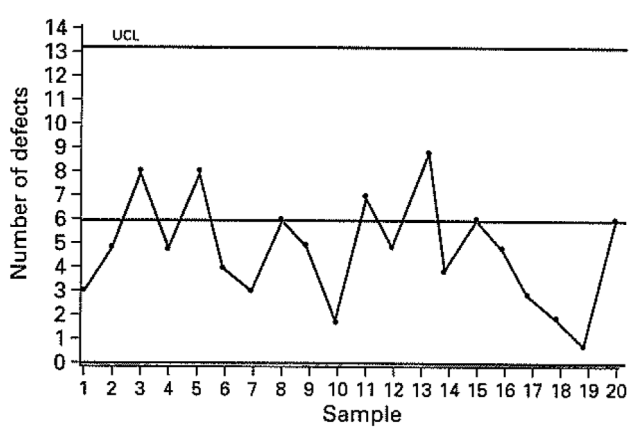


Figure 17.10 Additional production data for Example 17.5.