Kvantiler i t-fordelingen

$$P(T > t_{\alpha,\nu}) = \alpha$$

$\nu \backslash \alpha$	.150	.100	.075	.050	.025	.010	.005	.001	.0005
1	1.963	3.078	4.165	6.314	12.706	31.821	63.657	318.309	636.619
2	1.386	1.886	2.282	2.920	4.303	6.965	9.925	22.327	31.599
3	1.250	1.638	1.924	2.353	3.182	4.541	5.841	10.215	12.924
4	1.190	1.533	1.778	2.132	2.776	3.747	4.604	7.173	8.610
5	1.156	1.476	1.699	2.015	2.571	3.365	4.032	5.893	6.869
6	1.134	1.440	1.650	1.943	2.447	3.143	3.707	5.208	5.959
7	1.119	1.415	1.617	1.895	2.365	2.998	3.499	4.785	5.408
8	1.108	1.397	1.592	1.860	2.306	2.896	3.355	4.501	5.041
9	1.100	1.383	1.574	1.833	2.262	2.821	3.250	4.297	4.781
10	1.093	1.372	1.559	1.812	2.228	2.764	3.169	4.144	4.587
11	1.088	1.363	1.548	1.796	2.201	2.718	3.106	4.025	4.437
12	1.083	1.356	1.538	1.782	2.179	2.681	3.055	3.930	4.318
13	1.079	1.350	1.530	1.771	2.160	2.650	3.012	3.852	4.221
14	1.076	1.345	1.523	1.761	2.145	2.624	2.977	3.787	4.140
15	1.074	1.341	1.517	1.753	2.131	2.602	2.947	3.733	4.073
16	1.071	1.337	1.512	1.746	2.120	2.583	2.921	3.686	4.015
17	1.069	1.333	1.508	1.740	2.110	2.567	2.898	3.646	3.965
18	1.067	1.330	1.504	1.734	2.101	2.552	2.878	3.610	3.922
19	1.066	1.328	1.500	1.729	2.093	2.539	2.861	3.579	3.883
20	1.064	1.325	1.497	1.725	2.086	2.528	2.845	3.552	3.850

Measures of central tendency

Trimmed mean (TrMean)	A 5% trimmed mean is calculated. Minitab removes the smallest 5% and the largest 5% of the values (rounded to the nearest integer), and then calculates the mean of the remaining values.
Median	The median is in the middle of the data: half the observations are less than or equal to it, and half are greater than or equal to it.

For example, when N = 5 and you have data $x_{(1)}$, $x_{(2)}$, $x_{(3)}$, $x_{(4)}$, and $x_{(5)}$, the median = $x_{(3)}$.

When N = 6 and you have data $x_{(1)}$, $x_{(2)}$, $x_{(3)}$, $x_{(4)}$, $x_{(5)}$,and $x_{(6)}$:

$$\text{median} = \frac{(x_{(3)} + x_{(4)})}{2}$$

where $x_{(3)}$ and $x_{(4)}$ are the third and fourth observations.

Measures of position

First quartile (Q1)

Every group of data has three quartiles. If you sort the data from smallest to largest, the first 25% of the data is less than or equal to the first quartile.

To calculate quartiles, first order the data from smallest to largest. The first quartile (Q1) is the observation at position (N + 1) / 4, where N is the number of nonmissing observations. If the position is not an integer, interpolation is used.

For example, suppose N=10. Then (10 + 1)/4 = 2.75, and Q1 is between the second and third observations (call them $x_{(2)}$ and $x_{(3)}$), three-fourths of the way up.

Thus, $Q1 = x_{(2)} + 0.75(x_{(3)} - x_{(2)})$.

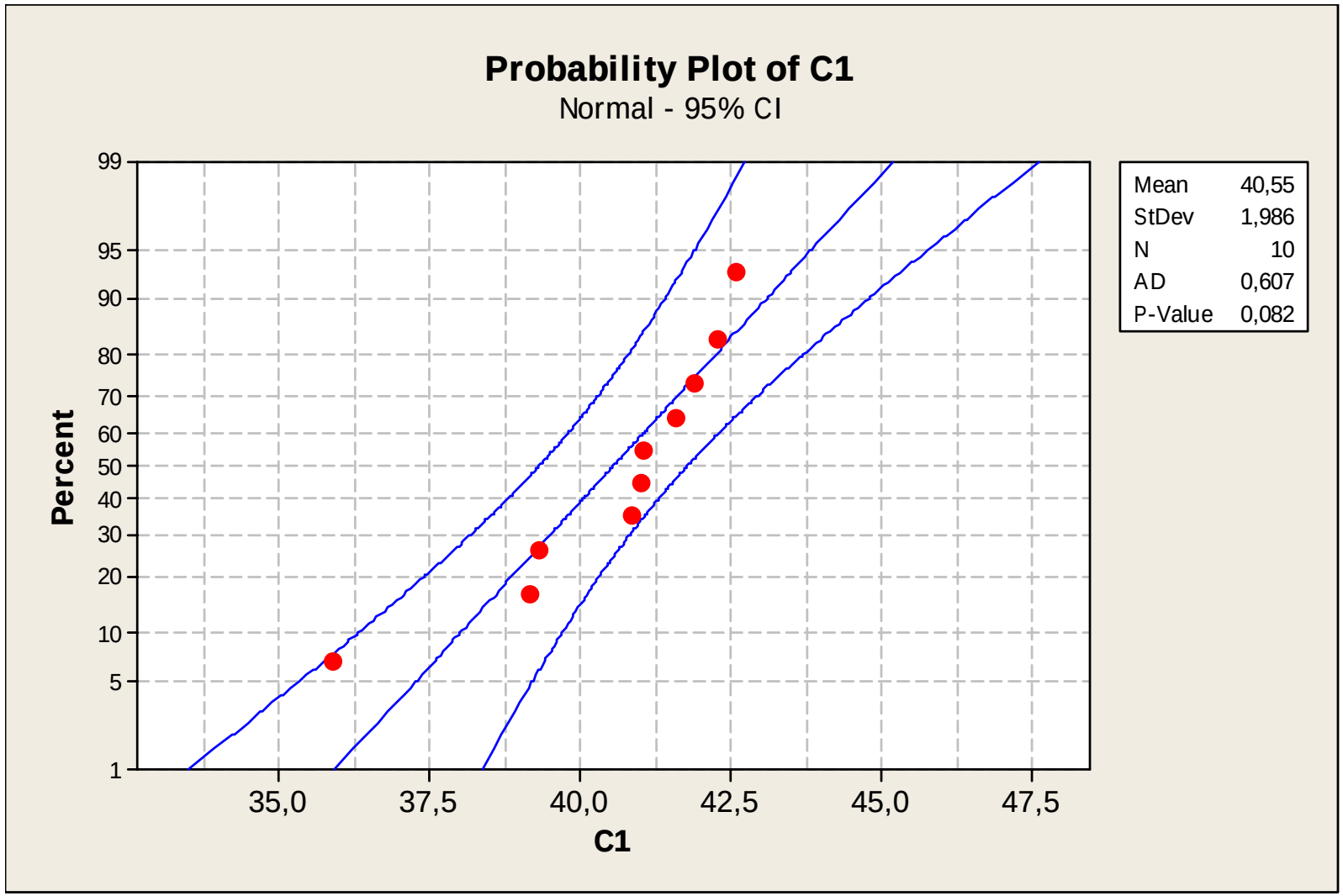
Third quartile (Q3)

Every group of data has three quartiles. If you sort the data from smallest to largest, the first 75% of the data is less than or equal to the third quartile and 25% of the data is greater than or equal to the third quartile.

To calculate quartiles, first order the data from smallest to largest. The third quartile (Q3) is the observation at position 3(N + 1) / 4, where N is the number of nonmissing observations. If the position is not an integer, interpolation is used.

For example, suppose N=10. Then 3(10 + 1)/4 = 8.25, $Q3 = x_{(8)} + 0.25(x_{(9)} - x_{(8)})$, where $x_{(8)}$ and $x_{(9)}$ are the eighth and ninth observations.

Normal-plott for Eksempel 3 (MINITAB 14)



Example 1: The industrial experment

An experiment was performed on a manufacturing plant by making in sequence 10 batches using the standard production method (A), followed by 10 batches of a modified method (B). The results from these trials are given in the table on the next slide. What evidence do the data provide that method B is better than method A?

SIF5068 Industriell statistikk, Høsten 2001

Eksempel 1: Industrieksempelet – bruk av referansefordelinger

I en kjemisk prosess har man i flere år benyttet en bestemt katalysator A. En ny type katalysator B som nylig er kommet ut på markedet er foreslått som et alternativ til A for å øke produksjonen. Vi ønsker å undersøke om den nye katalysatoren vil gi økt produksjon.

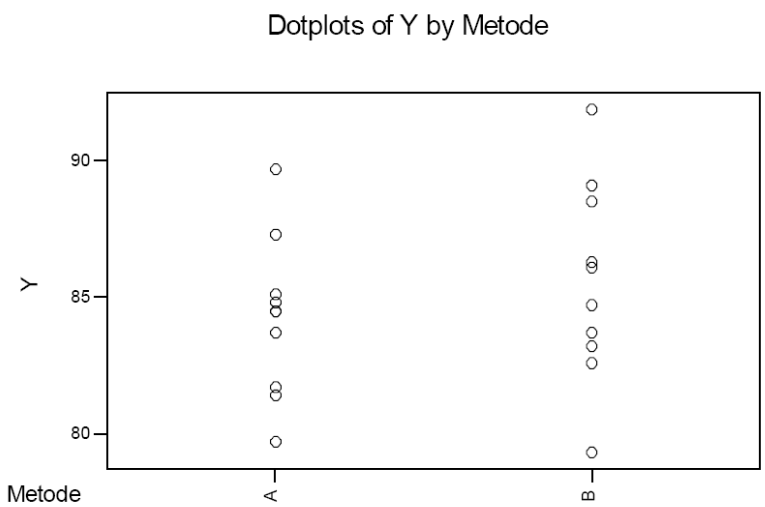
I et laboratorium blir det satt opp et forsøk for å sammenligne de to typene katalysatorer. Det blir gjort 10 observasjoner av produksjonen for hver av de to katalysatorene sekvensielt i tid.

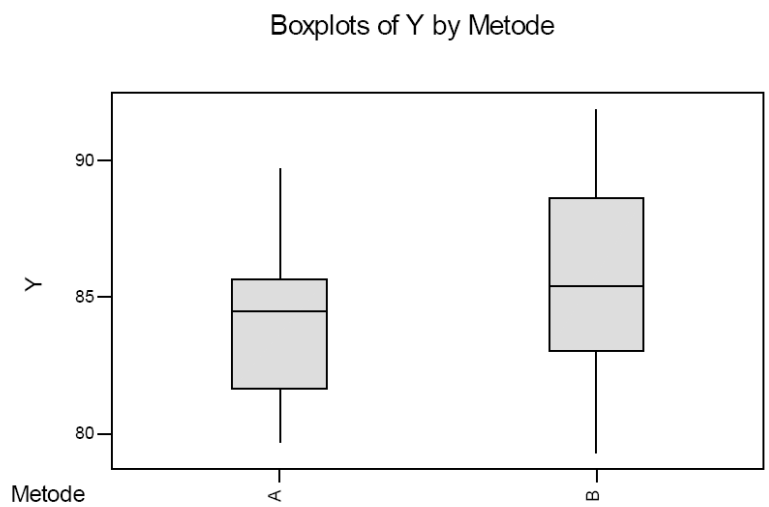
Tid	Metode	Y
1	A	89.7
2	A	81.4
3	A	84.5
4	A	84.8
5	A	87.3
6	A	79.7
7	A	85.1
8	A	81.7
9	A	83.7
10	A	84.5
11	B	84.7
12	B	86.1
13	B	83.2
14	B	91.9
15	B	86.3
16	B	79.3
17	B	82.6
18	B	89.1
19	B	83.7
20	B	88.5

Descriptive Statistics: Y by Metode

Variable	Metode	N	Mean	Median	TrMean	StDev
Y	A	10	84.24	84.50	84.13	2.90
	B	10	85.54	85.40	85.53	3.65

Variable	Metode	SE Mean	Min	Max	Q1	Q3
Y	A	0.92	79.70	89.70	81.63	85.65
	B	1.15	79.30	91.90	83.05	88.65


Dotplots of Y by Metode
Y
90
85
80
Metode
A
B


Boxplots of Y by Metode
Y
90
85
80
Metode
A
B

Antar lik varians: (Assuming equal variances:)

Two-Sample T-Test and CI: Y

```
Two-sample T for Y

M             N      Mean     StDev   SE Mean
1 (B)        10     85.54      3.65       1.2
2 (A)        10     84.24      2.90      0.92

Difference = mu (1) - mu (2)
Estimate for difference:  1.30
95% lower bound for difference: -1.26
T-Test of difference = 0 (vs >): T-Value = 0.88  P-Value = 0.195  DF = 18
Both use Pooled StDev = 3.30
```

Antar ikke lik varians: (Not assuming equal variances:)

Two-Sample T-Test and CI: Y

```
Two-sample T for Y

M             N      Mean     StDev   SE Mean
1 (B)        10     85.54      3.65       1.2
2 (A)        10     84.24      2.90      0.92

Difference = mu (B) - mu (A)
Estimate for difference:  1.30
95% lower bound for difference: -1.27
T-Test of difference = 0 (vs >): T-Value = 0.88  P-Value = 0.195  DF = 17
```

Test and confidence interval

Hypotheses

The hypotheses for a 2 Variance test are:

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$

where

- σ_1 = the standard deviation for the first population
- σ_2 = the standard deviation for the second population

Levene's test

The computational method for Levene's Test is a modification of Levene's procedure [9] developed by [2]. This method considers the distances of the observations from their sample median rather than their sample mean.

The test statistic is given by:

$$L = \frac{(N - k) \sum n_i (\overline{V}_{i\cdot} - \overline{V}_{\cdot\cdot})^2}{(k - 1) \sum\sum (\overline{V}_{ij} - \overline{V}_{i\cdot})^2}$$

where $V_{ij} = |X_{ij} - \tilde{X}_i|$, i = 1, ... , k, j = 1, ... , n_i and $\tilde{X}_i$ = median $\{X_{i1},...,X_{ini}\}$.

F test

$$F = \frac{S_1^2}{S_2^2}$$

where

- S_1^2 = the variance of the first sample
- S_2^2 = the variance of the second sample

Bonferroni confidence interval for variance

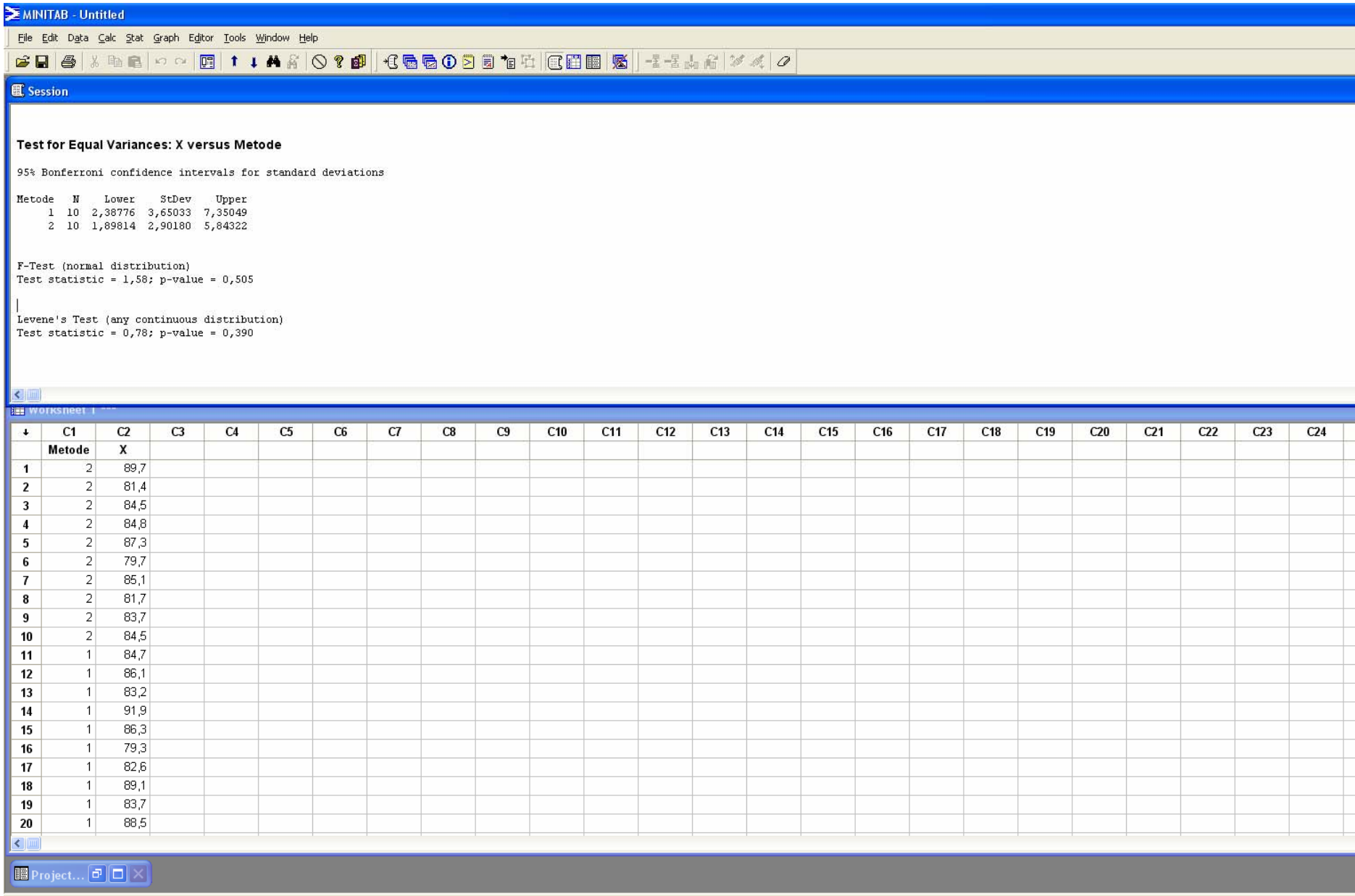
$$\frac{(n_i - 1) S_i^2}{\chi^2_{ni - 1,\ 1 - \alpha/2a}} \leq \sigma_i^2 \leq \frac{(n_i - 1) S_i^2}{\chi^2_{ni - 1,\ \alpha/2a}}$$

where

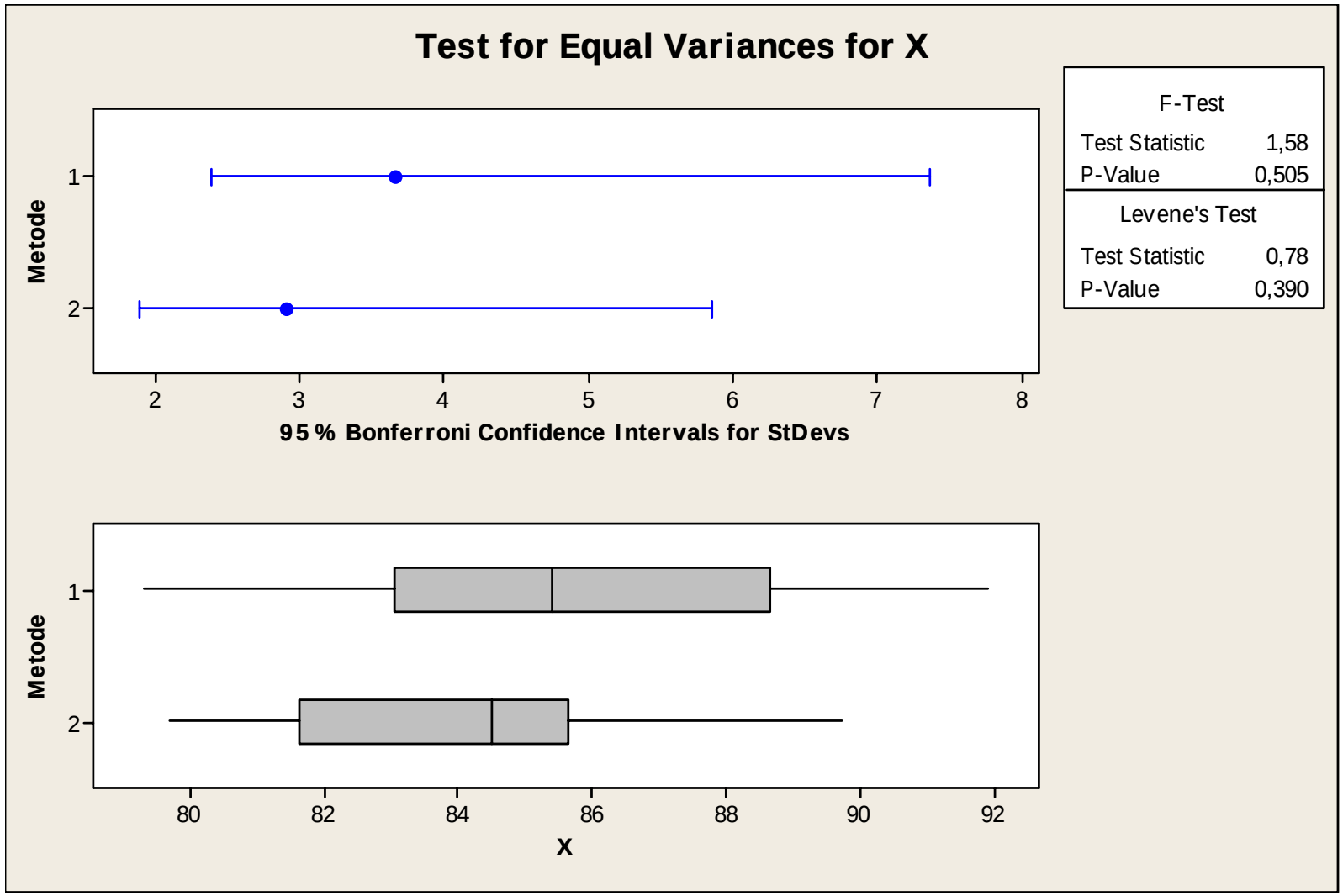
- n_i = the sample size of the ith level or category
- S_i^2 = the standard size of the ith level or category
- a = the number of levels or categories
- α = the level of significance

Back to top

The confidence interval for standard deviation is the square root of the two values.



	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24
	Metode	X																						
1	2	89,7																						
2	2	81,4																						
3	2	84,5																						
4	2	84,8																						
5	2	87,3																						
6	2	79,7																						
7	2	85,1																						
8	2	81,7																						
9	2	83,7																						
10	2	84,5																						
11	1	84,7																						
12	1	86,1																						
13	1	83,2																						
14	1	91,9																						
15	1	86,3																						
16	1	79,3																						
17	1	82,6																						
18	1	89,1																						
19	1	83,7																						
20	1	88,5																						


Test for Equal Variances for X
F-Test
Test Statistic 1,58
P-Value 0,505
Levene's Test
Test Statistic 0,78
P-Value 0,390
Metode
1
2
2
3
4
5
6
7
8
95% Bonferroni Confidence Intervals for StDevs
Metode
1
2
80
82
84
86
88
90
92
X

Kvantiler i Fisherfordelingen

$$P(F > f_{0.025,\nu_1,\nu_2}) = 0.025$$

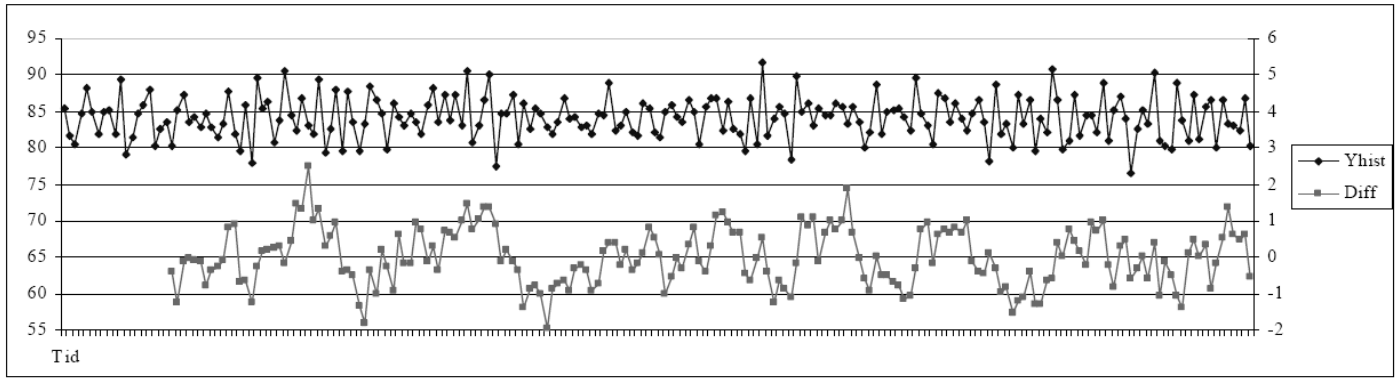
$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	9	10
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77

Reference distribution based on historical data:

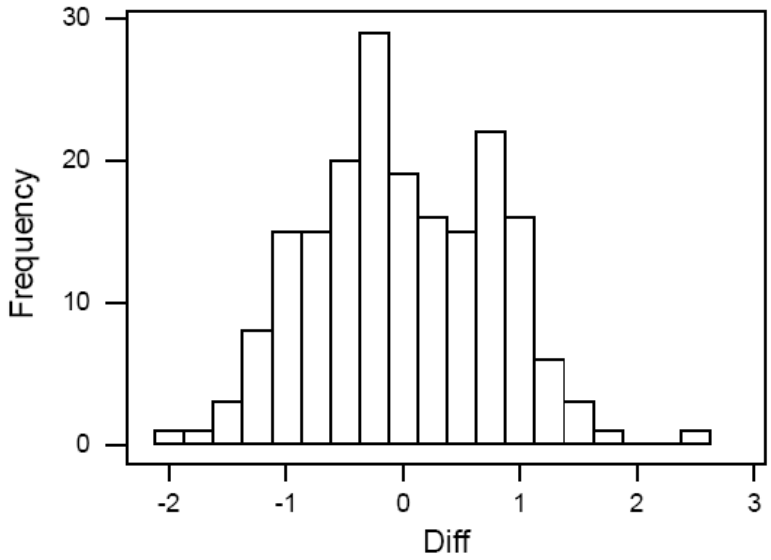
On the next slide are shown 210 earlier observations of the process for method A (upper curve) And the 191 differences between 10 and 10 succeeding averages. A table of the differences as well as a histogram are also given,

Referansefordeling basert på historiske data:

Nedenfor er vist plott av 210 tidligere observasjoner av prosessen for metode A, og de 191 differansene mellom 10 og 10 påfølgende gjennomsnitt. Tabell over differansene og et histogram av disse er også gitt.



-1.94	-1.81	-1.53	-1.40	-1.38	-1.33	-1.32	-1.30	-1.25	-1.24
-1.24	-1.20	-1.15	-1.10	-1.10	-1.07	-1.07	-1.06	-1.04	-1.02
-1.01	-0.97	-0.94	-0.93	-0.93	-0.91	-0.90	-0.90	-0.87	-0.86
-0.82	-0.81	-0.80	-0.79	-0.78	-0.76	-0.75	-0.68	-0.67	-0.66
-0.64	-0.64	-0.63	-0.62	-0.61	-0.60	-0.58	-0.58	-0.55	-0.53
-0.52	-0.52	-0.51	-0.50	-0.45	-0.45	-0.43	-0.43	-0.43	-0.42
-0.42	-0.40	-0.38	-0.37	-0.37	-0.36	-0.36	-0.36	-0.34	-0.32
-0.30	-0.30	-0.30	-0.30	-0.29	-0.27	-0.26	-0.23	-0.22	-0.21
-0.21	-0.18	-0.18	-0.17	-0.17	-0.17	-0.16	-0.16	-0.15	-0.15
-0.13	-0.13	-0.12	-0.12	-0.12	-0.11	-0.11	-0.10	-0.08	-0.05
-0.04	-0.03	-0.02	-0.01	-0.01	0.02	0.03	0.08	0.10	0.11
0.12	0.13	0.13	0.17	0.20	0.20	0.21	0.21	0.24	0.29
0.29	0.29	0.30	0.30	0.33	0.33	0.37	0.38	0.39	0.40
0.43	0.44	0.46	0.49	0.50	0.51	0.53	0.54	0.54	0.57
0.61	0.61	0.64	0.64	0.66	0.66	0.67	0.67	0.68	0.68
0.68	0.71	0.72	0.74	0.74	0.75	0.76	0.78	0.78	0.79
0.79	0.80	0.82	0.87	0.88	0.90	0.95	0.95	0.96	0.97
0.97	0.98	0.98	1.00	1.01	1.01	1.01	1.04	1.09	1.11
1.13	1.25	1.33	1.33	1.35	1.37	1.39	1.46	1.47	1.87
2.48									



Histogrammet vil nå uttrykke referansefordelingen basert på historiske data, og indikerer hvilke differanser som naturlig ville opptre dersom det ikke var forskjell mellom metode A og B. Ved opptelling finner vi at det er 9 differanser som er større enn verdien 1.3 som vi observerte i vårt eksperiment, og signifikanssannsynligheten blir da p-value = 9/191 = 0.047.

The histogram expresses the reference distribution based on historical data, and indicates which differences that naturally would be found if there was no difference between the methods A and B.

By a direct count we find that 9 differences exceed the value 1.3 which we observed in our experiment. The p-value is hence 9/191 = 0.047 (and there is thus reason to reject the hypothesis that the methods are equally good)