SOLUTION TMA 4255 May 26 2010

Problem 1

a) Residuals: • Normal plot, points are close to the line as they should be
- Residual histogram: OK since just 10 observ.
- Residuals vs. fitted values: Seems to be some ∪-shape, but OK since few observations.
- Residuals vs. order: This is essentially like a plot vs. x. No strong tendency so OK.

R^2 is 57.4% which shows that there is some noise in data, and 57.4% of variation is explained by model.

Conclusion: Both the functional form, the independence and normality are OK.

The p-value 0.050 is from testing

$H_0: \beta_1 = \beta_2 = 0$ vs. H_1: not both are 0.

b) $T_2 = \frac{\hat{\beta}_2}{SE(\hat{\beta}_2)} = -2.88$ Under $H_0: \beta_2 = 0$ is this

$t_{10-2-1} = t_7$.

We reject when $T_2 < -t_{0.05} \underset{\text{(table)}}{=} -1.895$

so we reject.

Since this is a one-sided test, the p-value is $0.024/2 = 0.012$

Will test $H_0: \beta_2 = -1.0$ vs. $H_1: \beta_2 \neq 1.0$.
Test statistic is then

$$T = \frac{\hat{\beta}_2 - (1.0)}{SE(\hat{\beta}_2)} = \frac{-1.1429 + 1.0}{SE(\hat{\beta}_2)} = -0.3603$$

so do not reject (crit. value for $|T|$ is $t_{7, 0.025} = 2.365$).

c) $S = 2.09859$

Conf. int. for σ^2: Use that $\frac{SSE}{\sigma^2} \sim \chi^2_7$

So: $P\left(1.690 < \frac{SSE}{\sigma^2} < 16.013\right) = 0.95$

$\Rightarrow P\left(\frac{SSE}{16.013} < \sigma^2 < \frac{SSE}{1.690}\right) = 0.95$

i.e. CI for σ^2: ~~(18.242~~ $(1.9252, 18.242)$

or CI for σ: $\underline{(1.3875, 4.2711)}$

$H_0: \sigma = \cancel{3}$ ~~is tested~~ vs. $H_1: \sigma \neq \cancel{3}$ is
tested with sign. level 0.05 (i.e. (1-0.95))
if ~~get~~ rejecting ~~accepting~~ when $\cancel{3} \notin$ interval. [So, ~~accept~~ reject here]

d) $f(x) = 10.1 + 7.36x - 1.14x^2$

$f'(x) = 7.36 - 2.28x = 0$

$$x = \frac{7.36}{2.28} = \underline{3.23}$$

Point prediction:

$$\hat{y}_0 = 10.100 + 7.357 \cdot 3.23 - 1.1429 \cdot 3.23^2$$
$$= 21.9393$$

$$SD(\hat{y}_0) = s \cdot \sqrt{x_0'(X'X)^{-1}x_0}$$

$$\text{so } \sqrt{x_0'(X'X)^{-1}x_0} = \frac{1.024}{2.09859} = 0.4879$$

$$\text{so } x_0'(X'X)^{-1}x_0 = 0.4879^2 = 0.2375$$

Thus P.I. is

$$(\hat{y}_0 \pm 2.365 \cdot 2.09859 \sqrt{1+0.2375}$$

$$21.9393 \pm 5.5212$$

$$(16.4181, 27.4605)$$

Problem 2:

a) Model: $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$

$i = 1,2,3$; $j = 1,2$ (Rose, Lemmon)

where μ is grand mean

α_i = effect of ~~...~~ supplement

β_j = effect of lake

γ_{ij} = interaction supplement i, lake j.

ε_{ijk} are independent $N(0, \sigma^2)$

$\sum \alpha_i = \sum \beta_j = \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$

Result: Testing for interaction

$H_0: \gamma_{ij} = 0$ ~~...~~ for all i,j

vs. not so.

Test statistic is $F = 2.71$ with $(2,6)$ df.

~~So does~~ P-value $= 0.145$, so we do not reject at sign. levels lower than this.

Having concluded that there is no interaction effect we consider main effects:

Supplement: $H_0: \alpha_i \equiv 0$ vs. H_1: not so.

Test-stat $F = 9.25$ at $(2,6)$ df,

so reject at all levels ≥ 0.015.

Lake is, however, not significant. So do not reject $H_0: \beta_j \equiv 0$ (~~...~~ very high P-value).

b) That SS(Supplement) is unchanged follows from their formulas.

That Total SS is unchanged since it just compares all the data to the grand ~~mean~~ average.

Thus
SS(Error) = 3123 − 1918.50 = 1204.50
Df(Suppl) = 3 − 1 = 2
Df(Total) = 12 − 1 = 11
so Df(Error) = 11 − 2 = 9

MS(Suppl) = 1918.50 / 2 = 959.25
MS(Error) = 1204.50 / 9 = 133.83

$$F = \frac{959.25}{133.83} = 7.1677$$

~~Df for F are (2, 9) so~~

F-test tests
H_0: ~~$\mu_1 = \mu_2 = \mu_3$~~ $\alpha_1 = \alpha_2 = \alpha_3 = 0$
(in the model where β_j and γ_{ij} are all set to 0).
i.e. tests $H_0: \mu_1 = \mu_2 = \mu_3 = 0$

The conclusion is to reject H_0 at all levels greater than P-value (approx 0.014)
Estimate of σ: $S = \sqrt{133.83} = 11.5685$.

Problem 3:

	Smok			
	Non	Mod	Heavy	
High	20 (33.24)	36 (30.31)	32 (24.44)	88
Normal	48 (34.76)	26 (31.69)	18 (25.56)	92
	68	62	50	180

H_0: independence vs H_1: not independence

Use independence test, χ^2-statistics

$df = (2-1) \times (3-1) = \underline{\underline{2}}$

$$\chi^2 = \frac{(20-33.24)^2}{33.24} + \frac{(36-30.31)^2}{30.31} + \dots$$

$= 16.982$, P-value $= 0.000$

Reject if $\chi^2 > 9.21$ so reject!

Problem 4:

Use C-chart

$X \sim$ Poisson (λ)

$\cancel{30:12} = 2$

When λ is known: $\lambda \pm 3\sqrt{\lambda}$

Must estimate λ by $\hat{\lambda} = \bar{X} = \frac{30}{12} = 2.5$

So: Use $2.5 \pm 3 \cdot \sqrt{2.5}$ i.e. $LCL = 0$, $UCL = 7.2434$

So: Seems to be OK.