TMA4275 LIFETIME ANALYSIS

Slides 1: Introduction

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After finishing this course you should ....

- know the most common concepts and distributions from lifetime modeling
- be able to use graphical methods for description and comparison of lifetime data
- be able to use statistical methods for statistical inference (estimation, confidence interval, hypothesis testing) of lifetime data
- be able to analyze lifetime data by using computer software (MINITAB)

LIFETIMES (WIDELY DEFINED)

Reliability engineering:

- Time to failure of a component or a system
- Number of cycles to failure (fatigue testing)
- Times between successive failures of a machine

Medical research:

- Time to death of a patient after start of certain treatment
- Time from entrance to discharge from a hospital
- Times between successive epileptic seizures for patient

Common technical definition of reliability:

The probability that a system or a component will perform its intended task, under given operational conditions, for a specified time period.

Lifetime (survival time) in medical research:

Time to occurrence of some event of interest for individuals in some population. The event may or may not be "death", and is often referred to as "failure".

WHY COLLECT AND ANALYZE LIFETIME/SURVIVAL/RELIABILITY DATA?

Reliability engineering:

- Assess reliability of a system/component/product
- Compare two or more products with respect to reliability
- Predict product reliability in the design phase
- Predict warranty claims for a product in the market

Medical research:

- Compare different treatments with respect to survival or recurrence
- Predict the outcome of an intervention or the life expectancy after the invention
- Identify risk factors for diseases and assess their magnitude

SPECIAL ASPECTS OF LIFETIME ANALYSIS IN STATISTICS

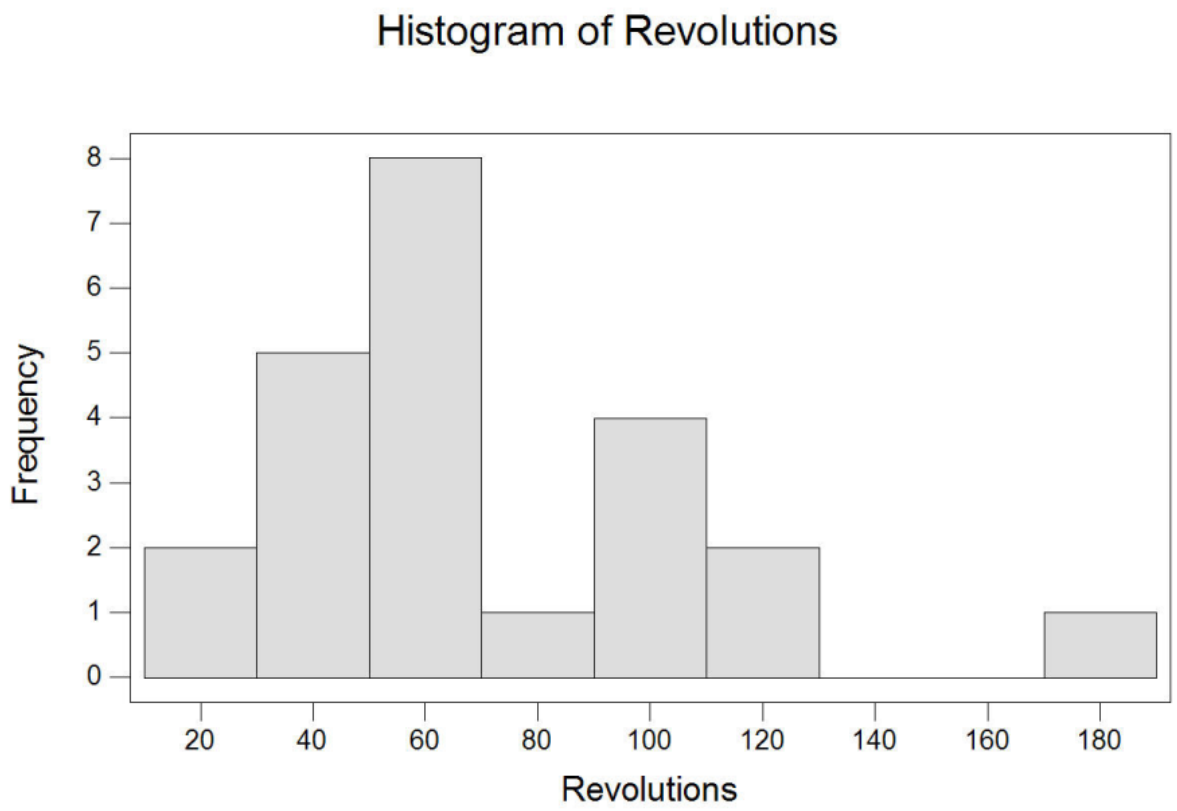
- Definition of starting time and failure time are difficult
- Definition of time scale (operation time, calendar time, number of cycles)
- Censored data (how can we use data from individuals or units for which the event of interest has not occurred within the observation period?)
- Effect of covariates (demographic, medical, environmental)
- What if an individual or unit dies or fails of another cause than the one we would like to study? ("competing risks")
- Recurrent events – what if the system can fail several times; how to analyze recurring stages of a disease?

BALL BEARING FAILURE DATA

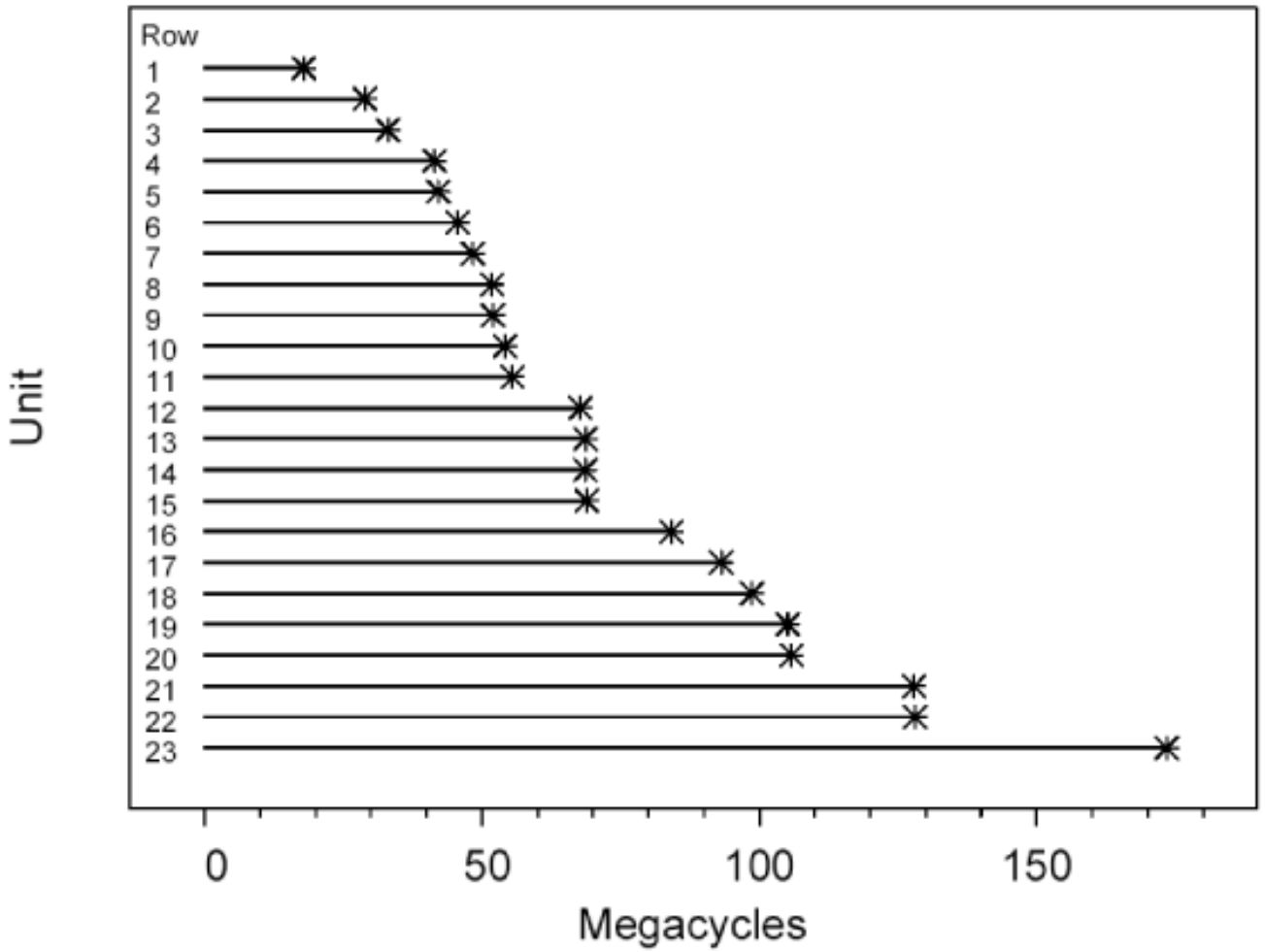
Data: Millions of revolutions to fatigue failure for 23 units

Question: How can we fit a parametric lifetime distribution to these data?

17,88	28,92	33,00	41,52	42,12	45,60	48,40	51,84
51,96	54,12	55,56	67,80	68,64	68,64	68,88	84,12
93,12	98,64	105,12	105,84	127,92	128,04	173,40	



BALL BEARING FAILURE DATA (EVENT PLOT)



IC DATA (MEEKER, 1987)

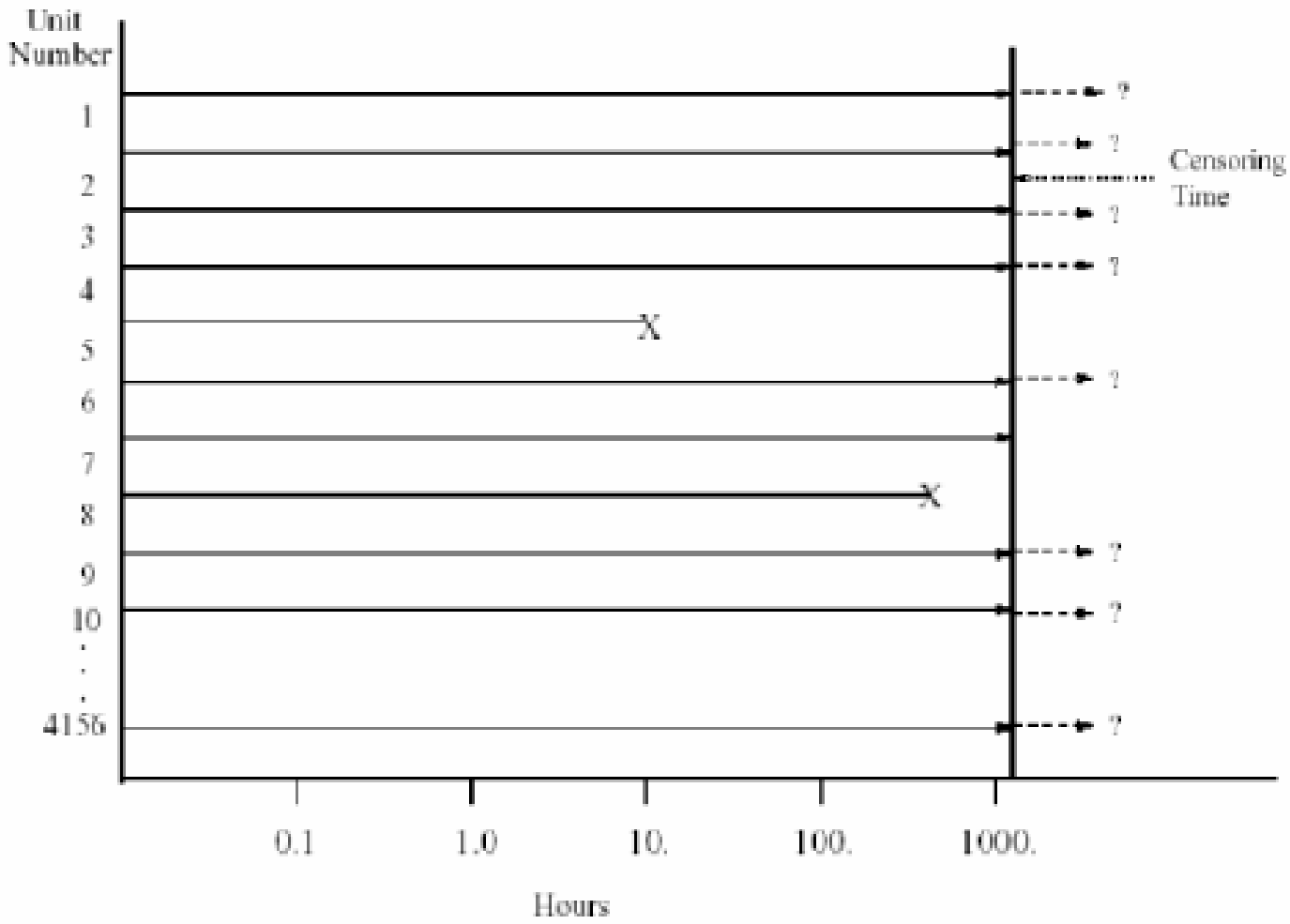
- Integrated circuit failure times in hours
  - n = 4156 ICs tested for 1,370 hours at 80° C and 80% relative humidity
  - There were 28 failures
  - When the test ended at 1,370 hours, 4128 units were still running

.10	.10	.15	.60	.80	.80
1.20	2.5	3.0	4.0	4.0	6.0
10.0	10.0	12.5	20.	20.	43.
43.	48.	48.	54.	74.	84.
94.	168.	263.	593.		

Questions of interest:

- How to estimate the distribution of the failure time when there are censored observations?
- Probability of failure before 100 hours?
- Failure rate by 100 hours?
- Proportion failed after 10^5 hours?

IC DATA (EVENT PLOT)



SURVIVAL OF MULTIPLE MYELOMA PATIENTS

- *Multiple myeloma* is a malignant disease characterised by the accumulation of abnormal plasma cells, a type of white blood cell, in the bone marrow.
- Data (next slide) from Medical Center of the University of West Virginia, USA.
- **Aim:** To examine the association between certain explanatory variables or covariates and the survival time of patients in months from diagnosis until death from multiple myeloma).

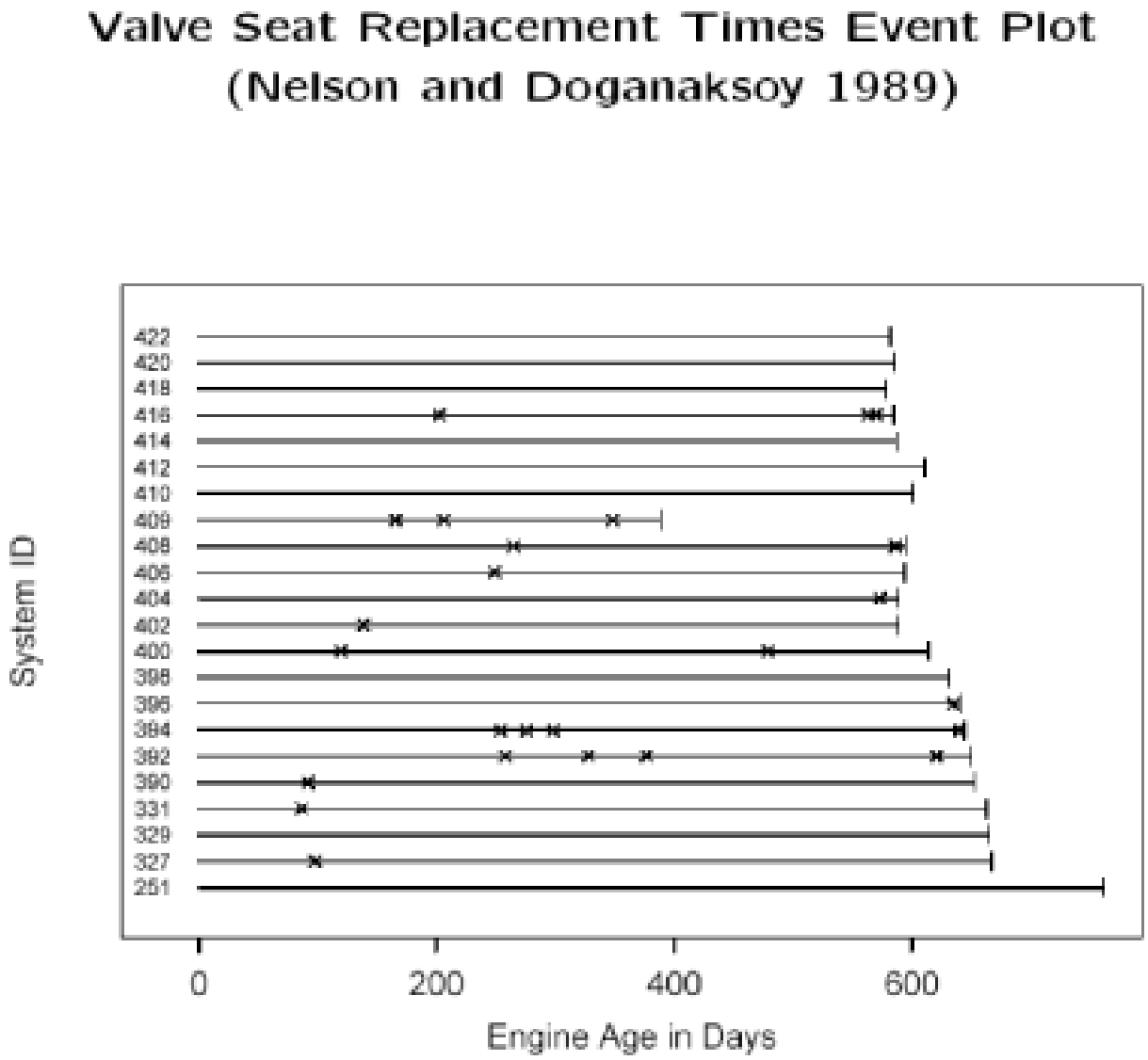
Table 1.3 *Survival times of patients in a study on multiple myeloma.*

Patient number	Survival time	Status	Age	Sex	*Bun*	*Ca*	*Hb*	*Pcells*	*Protein*
1	13	1	66	1	25	10	14.6	18	1
2	52	0	66	1	13	11	12.0	100	0
3	6	1	53	2	15	13	11.4	33	1
4	40	1	69	1	10	10	10.2	30	1
5	10	1	65	1	20	10	13.2	66	0
6	7	0	57	2	12	8	9.9	45	0
7	66	1	52	1	21	10	12.8	11	1
8	10	0	60	1	41	9	14.0	70	1
9	10	1	70	1	37	12	7.5	47	0
10	14	1	70	1	40	11	10.6	27	0
11	16	1	68	1	39	10	11.2	41	0
12	4	1	50	2	172	9	10.1	46	1
13	65	1	59	1	28	9	6.6	66	0
14	5	1	60	1	13	10	9.7	25	0
15	11	0	66	2	25	9	8.8	23	0
16	10	1	51	2	12	9	9.6	80	0
17	15	0	55	1	14	9	13.0	8	0
18	5	1	67	2	26	8	10.4	49	0
19	76	0	60	1	12	12	14.0	9	0
20	56	0	66	1	18	11	12.5	90	0

Problem 2

A clinical trial to evaluate the efficacy of chemotherapy for a specific cancer was conducted. After reaching a state of remission (disappearance of cancer) through treatment, the patients who entered the study were randomized into two groups. The first group received maintenance chemotherapy, the second (or control) group did not. For a preliminary analysis during the course of the trial the data were as follows:
Length of complete remission (in weeks).

Maintenance group: 9, 13, 13^+, 23, 24^+, 34, 45^+, 55, 161^+
Control group: 5, 13, 13, 16^+, 20, 21, 43, 45
$+$ indicates censored observation.


Valve Seat Replacement Times Event Plot
(Nelson and Doganaksoy 1989)
System ID
422
420
418
416
414
412
410
409
408
406
404
402
400
398
396
394
392
390
331
329
327
251
0
200
400
600
Engine Age in Days

VALVE SEAT REPLACEMENT DATA

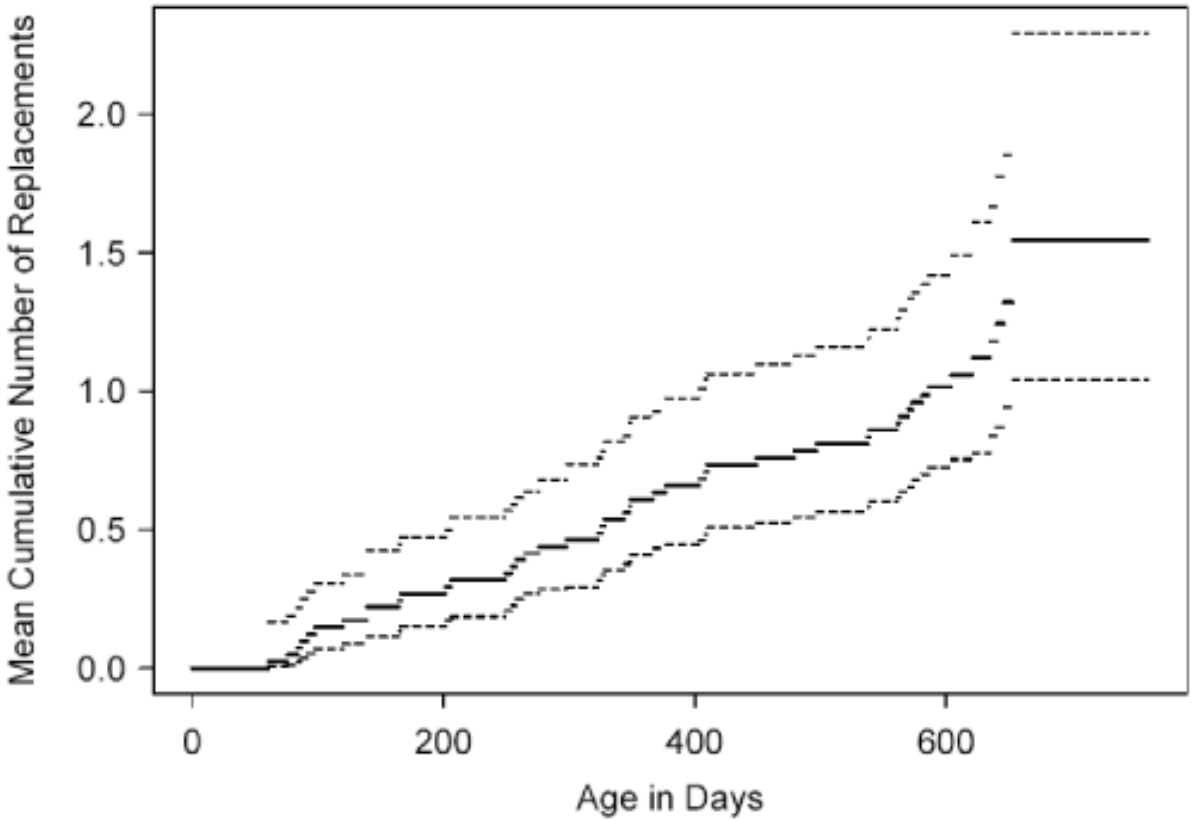
Data on previous slide are collected from valve seats from a fleet of 41 diesel engines. Each engine has 16 valves. (Time unit is days of operation).

Questions of interest:

- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

ESTIMATED NUMBER OF VALVE SEAT REPLACEMENTS

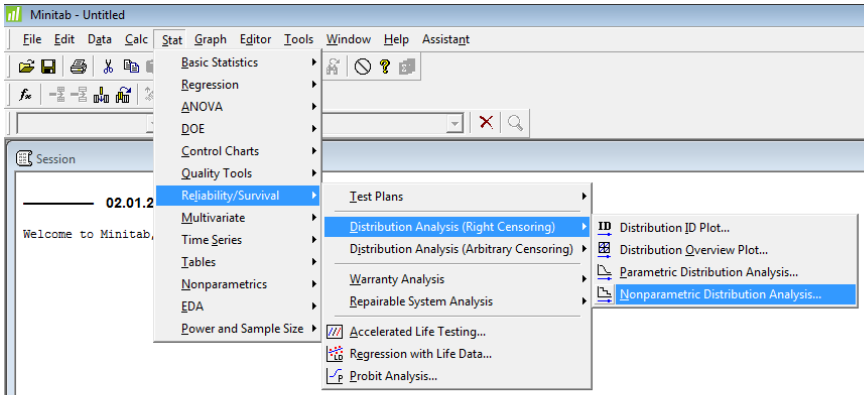
- Middle curve is cumulative estimated number of replacements for one engine, as a function of age.
- Lower and upper curves are 95% confidence limits.



MINITAB EXAMPLE
- THE DATA WORKSHEET

C1: Lifetimes

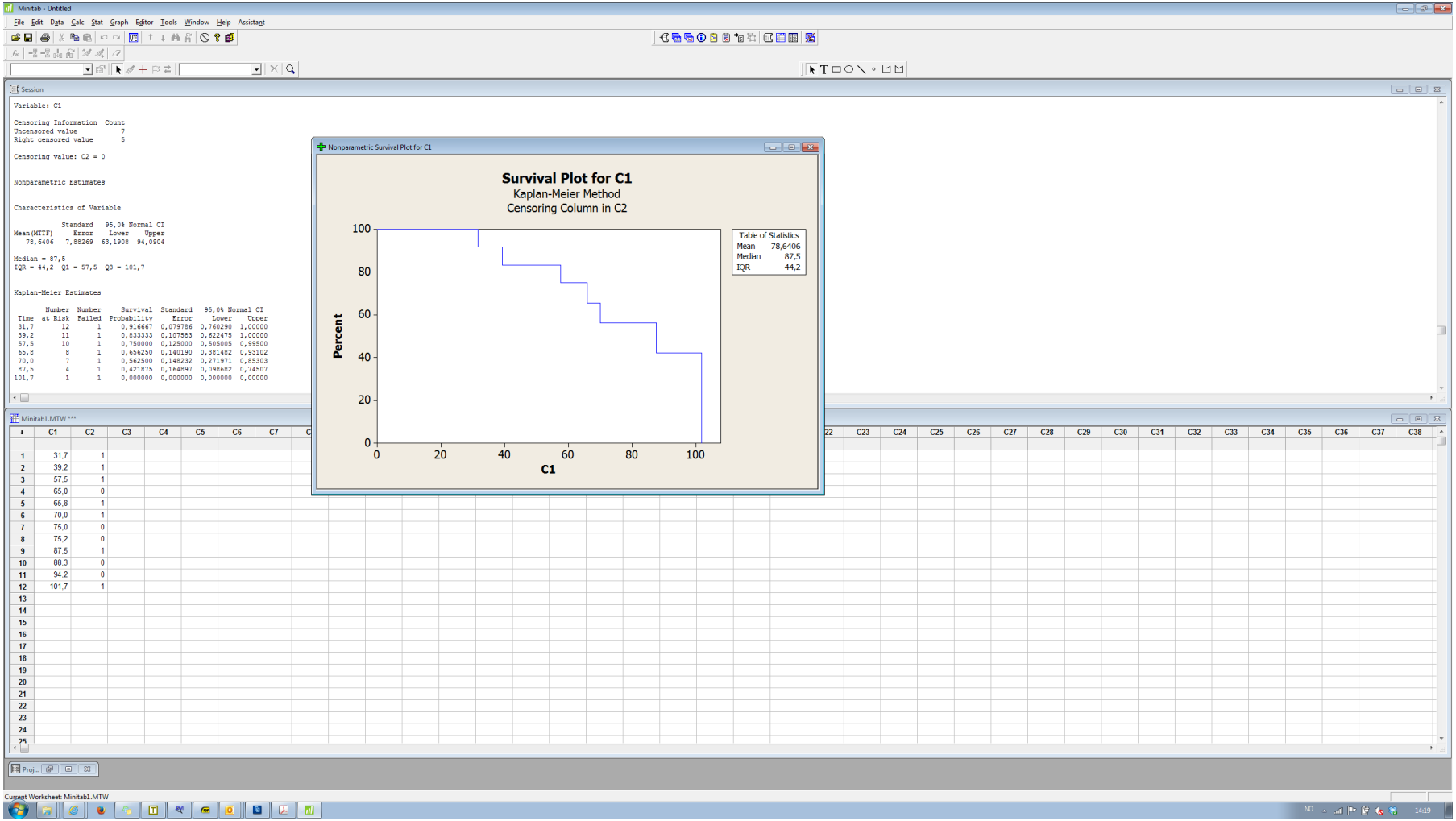
C2: Censoring indicators (0 = censored)



Minitab1.MTW ***

	C1	C2
1	31,7	1
2	39,2	1
3	57,5	1
4	65,0	0
5	65,8	1
6	70,0	1
7	75,0	0
8	75,2	0
9	87,5	1
10	88,3	0
11	94,2	0
12	101,7	1
13		
14		

MINITAB EXAMPLE
- ESTIMATION OF SURVIVAL FUNCTION



MORTALITY TABLE - DEATH HAZARD BY AGE

Statistisk sentralbyrå
Statistics Norway

Tabell

Lukk tabell | Last ned tabellen i Excel | Last ned tabellen som CSV-fil

Dødelighetstabeller

	2012											
	Levende ved alder x, lx			Døde i alder x til x+1, dx			Forventet gjenstående levetid ved alder x, ex			Dødssannsynlighet for alder x (Promille) (Uglattet), qx		
	Begge kjønn	Menn	Kvinner	Begge kjønn	Menn	Kvinner	Begge kjønn	Menn	Kvinner	Begge kjønn	Menn	Kvinner
0 år	100 000	100 000	100 000	248	280	214	81,45	79,42	83,41	2,479	2,798	2,143
1 år	99 752	99 720	99 786	27	25	30	80,65	78,64	82,59	0,274	0,251	0,299
2 år	99 725	99 695	99 756	11	9	13	79,67	77,66	81,61	0,110	0,092	0,129
3 år	99 714	99 686	99 743	11	15	6	78,68	76,67	80,62	0,110	0,153	0,065
4 år	99 703	99 671	99 737	5	0	10	77,69	75,68	79,63	0,048	0,000	0,099
5 år	99 698	99 671	99 727	15	22	7	76,69	74,68	78,64	0,146	0,221	0,066
6 år	99 683	99 649	99 720	2	0	3	75,70	73,70	77,64	0,016	0,000	0,033
7 år	99 682	99 649	99 717	7	6	7	74,70	72,70	76,65	0,066	0,065	0,068
8 år	99 675	99 642	99 710	10	10	10	73,71	71,70	75,65	0,099	0,097	0,102
9 år	99 665	99 633	99 700	8	3	14	72,71	70,71	74,66	0,084	0,033	0,137
10 år	99 657	99 629	99 686	13	13	14	71,72	69,71	73,67	0,134	0,132	0,137
11 år	99 644	99 616	99 673	11	10	13	70,73	68,72	72,68	0,114	0,095	0,133
12 år	99 632	99 607	99 659	8	9	7	69,74	67,73	71,69	0,079	0,093	0,065
13 år	99 624	99 597	99 653	13	6	20	68,74	66,73	70,69	0,127	0,062	0,196
14 år	99 612	99 591	99 633	16	21	10	67,75	65,74	69,71	0,158	0,216	0,097
15 år	99 596	99 570	99 624	15	18	13	66,76	64,75	68,71	0,154	0,180	0,127
16 år	99 581	99 552	99 611	24	27	22	65,77	63,76	67,72	0,245	0,267	0,221
17 år	99 556	99 525	99 589	18	21	16	64,79	62,78	66,74	0,185	0,209	0,159
18 år	99 538	99 504	99 573	34	53	13	63,80	61,79	65,75	0,339	0,537	0,127
19 år	99 504	99 451	99 560	21	29	13	62,82	60,82	64,76	0,214	0,295	0,126
20 år	99 483	99 422	99 548	48	66	28	61,84	59,84	63,76	0,480	0,667	0,279
21 år	99 435	99 355	99 520	46	69	21	60,86	58,88	62,78	0,459	0,694	0,212
22 år	99 389	99 286	99 499	47	72	21	59,89	57,92	61,79	0,473	0,726	0,211
23 år	99 342	99 214	99 478	30	49	9	58,92	56,96	60,81	0,298	0,499	0,091

MORTALITY TABLE - DEATH HAZARD BY AGE

72 år	82 194	78 643	85 922	1 676	1 970	1 387	14,24	12,96	15,27	20,390	25,047	16,139
73 år	80 518	76 673	84 535	1 659	1 953	1 377	13,53	12,28	14,52	20,609	25,468	16,285
74 år	78 859	74 720	83 158	1 765	1 952	1 592	12,80	11,59	13,75	22,386	26,124	19,140
75 år	77 094	72 768	81 567	2 044	2 365	1 746	12,08	10,89	13,01	26,516	32,494	21,409
76 år	75 050	70 404	79 820	2 046	2 343	1 779	11,40	10,24	12,28	27,262	33,274	22,293
77 år	73 004	68 061	78 041	2 381	2 794	2 014	10,70	9,57	11,55	32,611	41,045	25,807
78 år	70 623	65 268	76 027	2 716	3 088	2 385	10,05	8,96	10,84	38,459	47,307	31,370
79 år	67 907	62 180	73 642	2 891	3 405	2 434	9,43	8,38	10,18	42,567	54,768	33,050
80 år	65 016	58 775	71 208	2 997	3 420	2 638	8,83	7,84	9,51	46,098	58,195	37,045
81 år	62 019	55 354	68 570	3 398	3 783	3 082	8,23	7,29	8,86	54,791	68,343	44,952
82 år	58 621	51 571	65 488	3 661	4 101	3 298	7,68	6,79	8,25	62,457	79,529	50,365
83 år	54 960	47 470	62 189	4 013	4 211	3 887	7,15	6,33	7,66	73,026	88,712	62,505
84 år	50 946	43 259	58 302	3 867	4 172	3 667	6,68	5,90	7,14	75,895	96,434	62,888
85 år	47 080	39 087	54 636	3 997	4 117	3 965	6,19	5,48	6,58	84,906	105,321	72,570
86 år	43 082	34 970	50 671	4 236	4 079	4 441	5,71	5,06	6,06	98,324	116,649	87,642
87 år	38 846	30 891	46 230	4 213	4 096	4 399	5,28	4,67	5,59	108,449	132,591	95,146
88 år	34 633	26 795	41 831	4 292	3 894	4 735	4,86	4,30	5,13	123,925	145,341	113,199
89 år	30 341	22 901	37 096	4 114	3 876	4 442	4,48	3,95	4,72	135,592	169,266	119,733
90 år	26 227	19 024	32 655	4 367	3 703	5 047	4,10	3,65	4,29	166,501	194,620	154,562
91 år	21 861	15 322	27 607	3 650	3 069	4 244	3,82	3,41	3,98	166,963	200,273	153,723
92 år	18 211	12 253	23 363	3 732	2 932	4 485	3,49	3,14	3,62	204,937	239,318	191,977
93 år	14 479	9 321	18 878	3 015	2 318	3 665	3,26	2,97	3,36	208,220	248,707	194,145
94 år	11 464	7 003	15 213	2 698	1 823	3 458	2,99	2,79	3,05	235,366	260,306	227,292
95 år	8 766	5 180	11 755	2 301	1 559	2 958	2,75	2,60	2,80	262,529	300,960	251,671
96 år	6 464	3 621	8 797	1 871	1 134	2 496	2,55	2,51	2,57	289,392	313,095	283,759
97 år	4 594	2 487	6 301	1 432	795	1 952	2,39	2,42	2,39	311,787	319,717	309,819
98 år	3 161	1 692	4 349	1 071	646	1 426	2,25	2,32	2,24	338,639	381,843	327,848
99 år	2 091	1 046	2 923	711	316	1 017	2,14	2,45	2,08	340,030	302,252	347,939
100 år	1 380	730	1 906	495	209	711	1,99	2,30	1,93	358,820	286,341	373,123
101 år	885	521	1 195	344	172	479	1,82	2,02	1,78	388,786	329,680	401,004
102 år	541	349	716	234	90	331	1,66	1,76	1,63	432,013	258,075	462,088
103 år	307	259	385	155	148	189	1,54	1,20	1,60	504,285	571,937	491,822
104 år	152	111	196	55	70	61	1,59	1,14	1,67	363,091	632,121	313,445
105 år	97	41	134	28	11	40	1,21	1,24	1,20	290,260	264,859	295,930
106 år	69	30	95	35	17	47	0,50	0,50	0,50	501,251	550,671	494,864

Statistikkbanken kildetabell 07902