# TMA4275 LIFETIME ANALYSIS

## Slides 16-DRAFT: Parametric estimation in NHPPs

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## Simple Example With 3 Systems

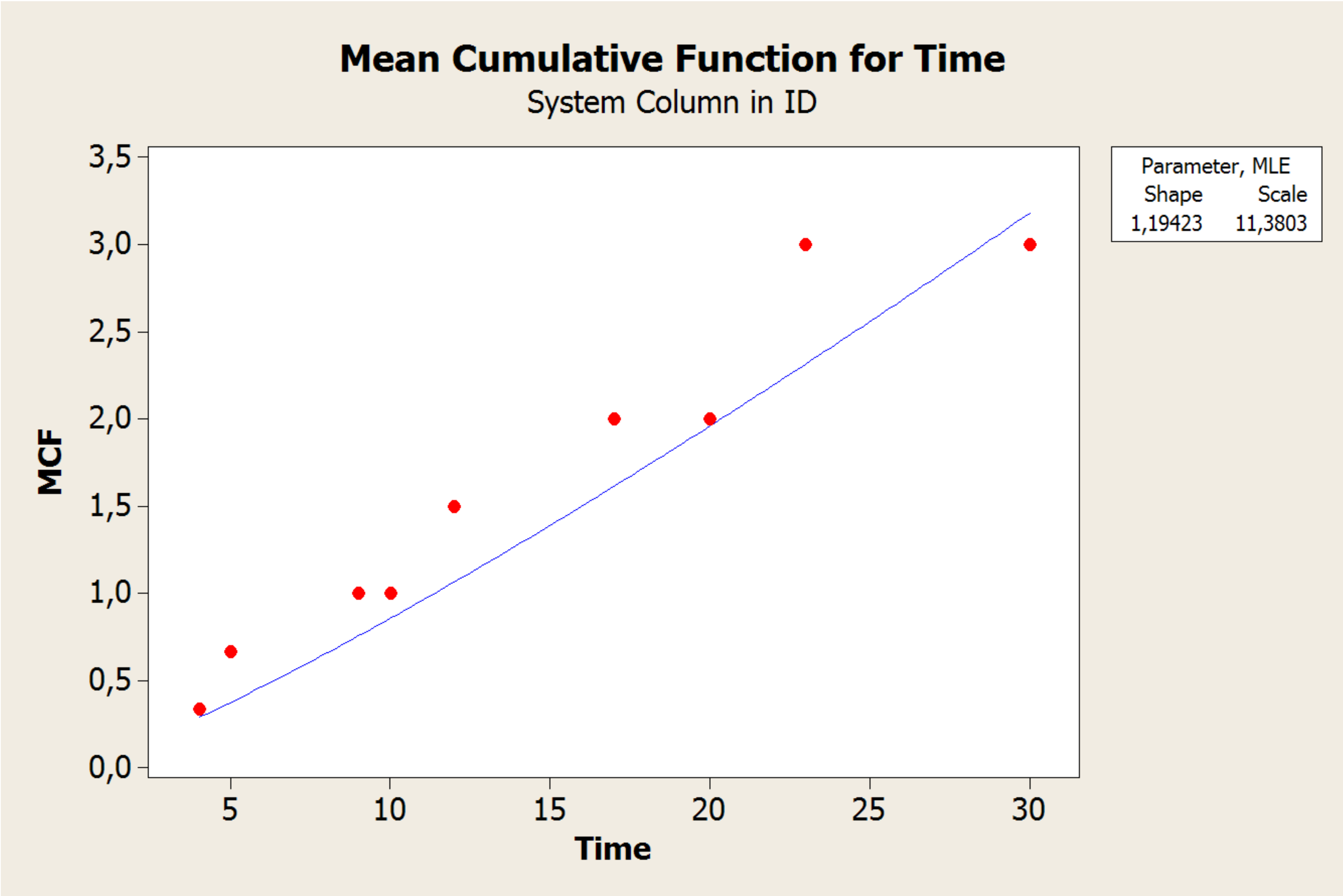
## Power Law NHPP Model: $W(t;\alpha,\theta) = (t/\theta)^{\alpha}$

**Results for: SimpleNHPP.MTW**

**Parametric Growth Curve: Time**

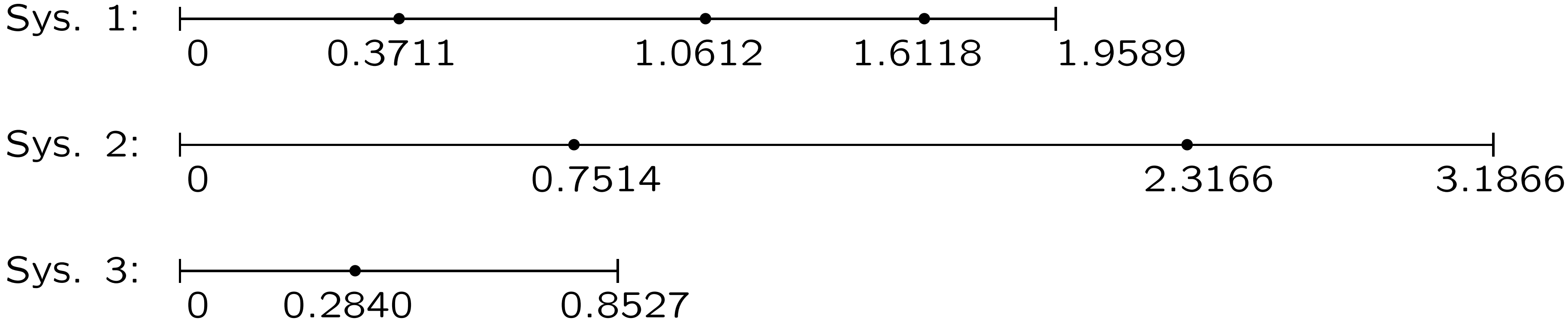
```
System:  ID

Model: Power-Law Process
Estimation Method: Maximum Likelihood


Parameter Estimates

                     Standard    95% Normal CI
Parameter  Estimate     Error     Lower     Upper
Shape       1,19423     0,445  0,323015   2,06545
Scale       11,3803     4,840   1,89335   20,8672


Test for Equal Shape Parameters
Bartlett's Modified Likelihood Ratio Chi-Square

Test Statistic   0,06
P-Value         0,972
DF                  2


Trend Tests

                   MIL-Hdbk-189        Laplace's
                TTT-based  Pooled  TTT-based  Pooled  Anderson-Darling
Test Statistic       9,03    8,89       0,28    0,31              0,28
P-Value             0,599   0,576      0,781   0,756             0,954
DF                     12      12
```

Simple Example With 3 Systems

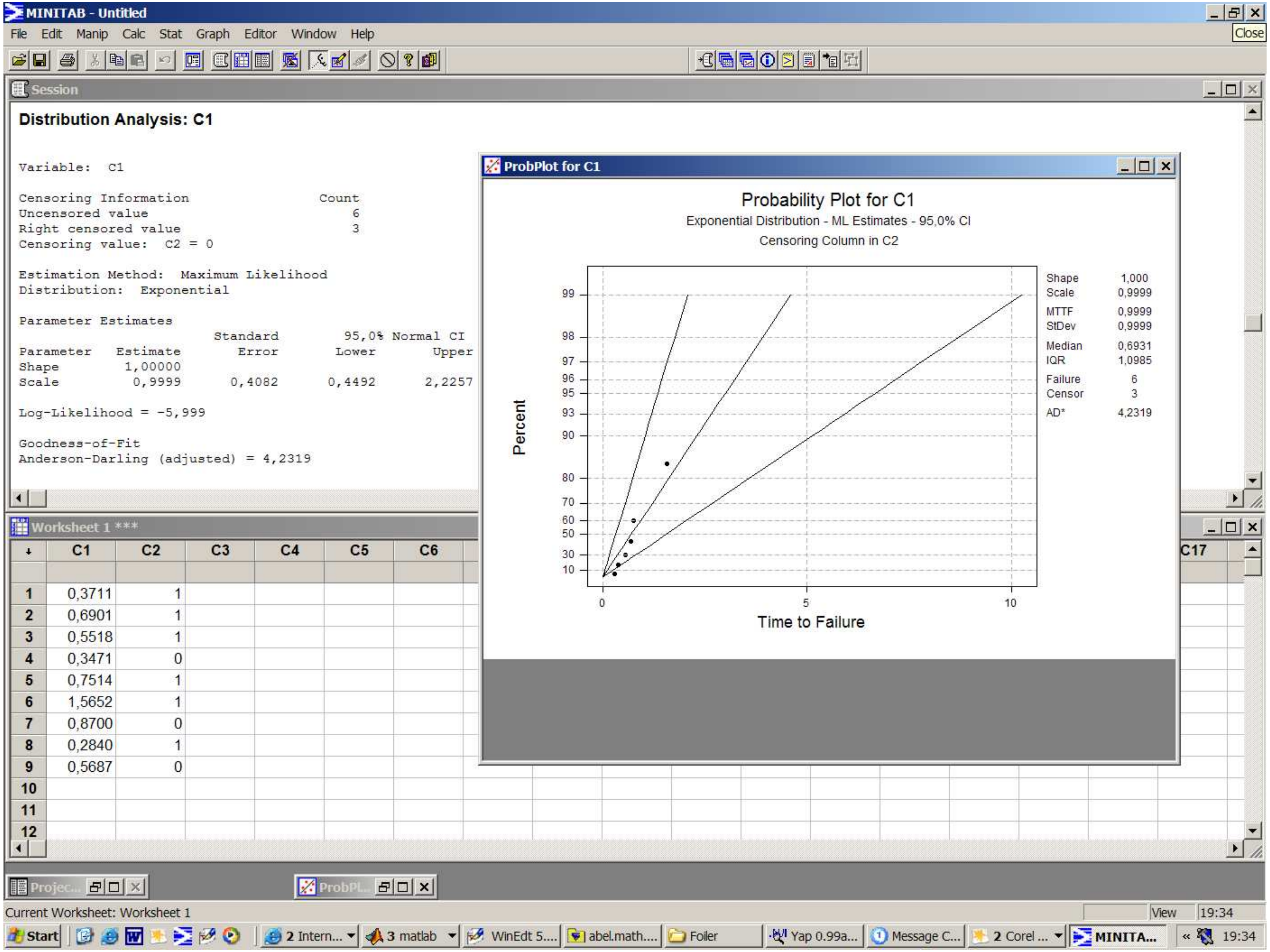


RESIDUAL PROCESS: "SIMPLE EXAMPLE".
Data points (and endpoints on axes) are transformed with the estimated cumulative ROCOF,

$$\widehat{W}(t) = 0.0538 \cdot t^{1.20}$$




Times between events, plus censored times at the end of each axis, are on the next slide anlysed by MINITAB as a set of censored exponential variables.


MINITAB - Untitled
File Edit Manip Calc Stat Graph Editor Window Help
Session
Distribution Analysis: C1
Variable: C1
Censoring Information Count
Uncensored value 6
Right censored value 3
Censoring value: C2 = 0
Estimation Method: Maximum Likelihood
Distribution: Exponential
Parameter Estimates
Standard 95,0% Normal CI
Parameter Estimate Error Lower Upper
Shape 1,00000
Scale 0,9999 0,4082 0,4492 2,2257
Log-Likelihood = -5,999
Goodness-of-Fit
Anderson-Darling (adjusted) = 4,2319
ProbPlot for C1
Probability Plot for C1
Exponential Distribution - ML Estimates - 95,0% CI
Censoring Column in C2
Percent
Time to Failure
Shape 1,000
Scale 0,9999
MTTF 0,9999
StDev 0,9999
Median 0,6931
IQR 1,0985
Failure 6
Censor 3
AD* 4,2319
Worksheet 1 ***
C1 C2 C3 C4 C5 C6 C17
1 0,3711 1
2 0,6901 1
3 0,5518 1
4 0,3471 0
5 0,7514 1
6 1,5652 1
7 0,8700 0
8 0,2840 1
9 0,5687 0
Current Worksheet: Worksheet 1

## Valve Seat Replacement Times (Nelson and Doganaksoy 1989)

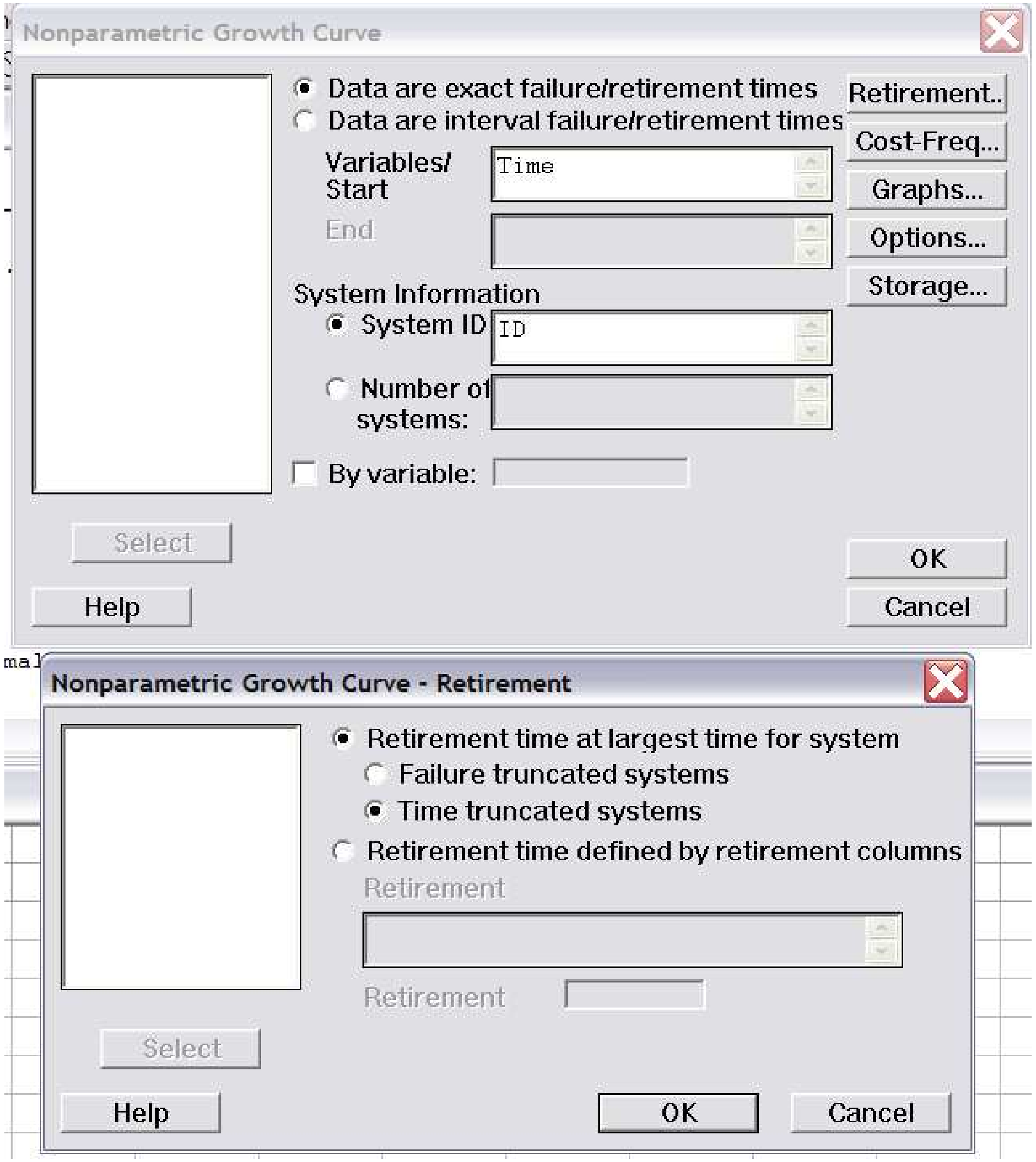
Data collected from valve seats from a fleet of 41 diesel engines (days of operation)

- Each engine has 16 valves
- Does the replacement rate increase with age?
- How many replacement valves will be needed in the future?
- Can valve life in these systems be modeled as a renewal process?

# VALVESEAT DATA

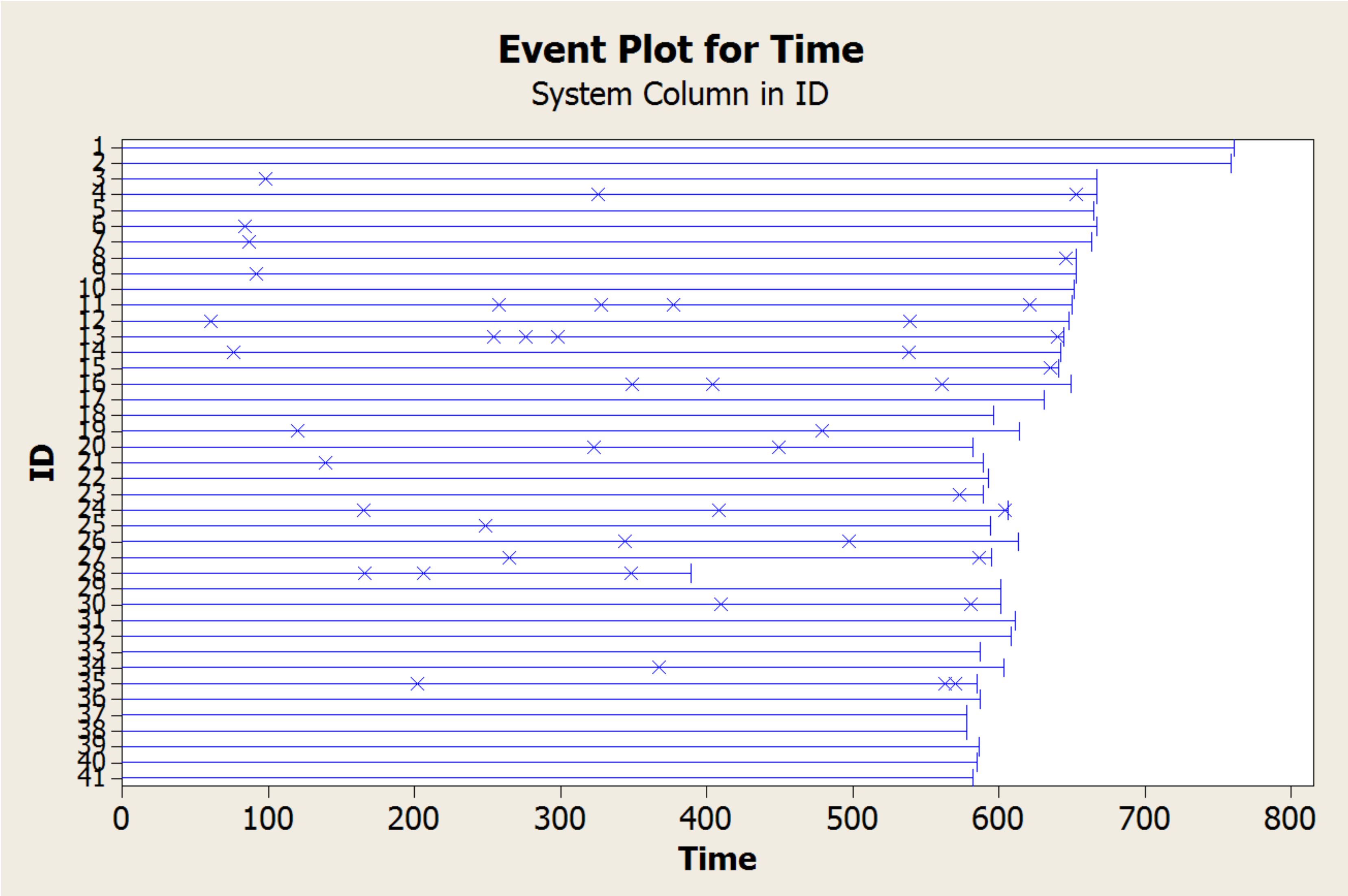
TMA4275valveseat.MTW ***

| ↓ | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C |
|---|---|---|---|---|---|---|---|---|---|
| | ID | Time | | | | | | | |
| 1 | 1 | 761 | | | | | | | |
| 2 | 2 | 759 | | | | | | | |
| 3 | 3 | 98 | | | | | | | |
| 4 | 3 | 667 | | | | | | | |
| 5 | 4 | 326 | | | | | | | |
| 6 | 4 | 653 | | | | | | | |
| 7 | 4 | 653 | | | | | | | |
| 8 | 4 | 667 | | | | | | | |
| 9 | 5 | 665 | | | | | | | |
| 10 | 6 | 84 | | | | | | | |
| 11 | 6 | 667 | | | | | | | |
| 12 | 7 | 87 | | | | | | | |
| 13 | 7 | 663 | | | | | | | |
| 14 | 8 | 646 | | | | | | | |
| 15 | 8 | 653 | | | | | | | |
| 16 | 9 | 92 | | | | | | | |
| 17 | 9 | 653 | | | | | | | |
| 18 | 10 | 651 | | | | | | | |
| 19 | 11 | 258 | | | | | | | |
| 20 | 11 | 328 | | | | | | | |
| 21 | 11 | 377 | | | | | | | |
| 22 | 11 | 621 | | | | | | | |
| 23 | 11 | 650 | | | | | | | |
| 24 | 12 | 61 | | | | | | | |
| 25 | 12 | 539 | | | | | | | |
| 26 | 12 | 648 | | | | | | | |

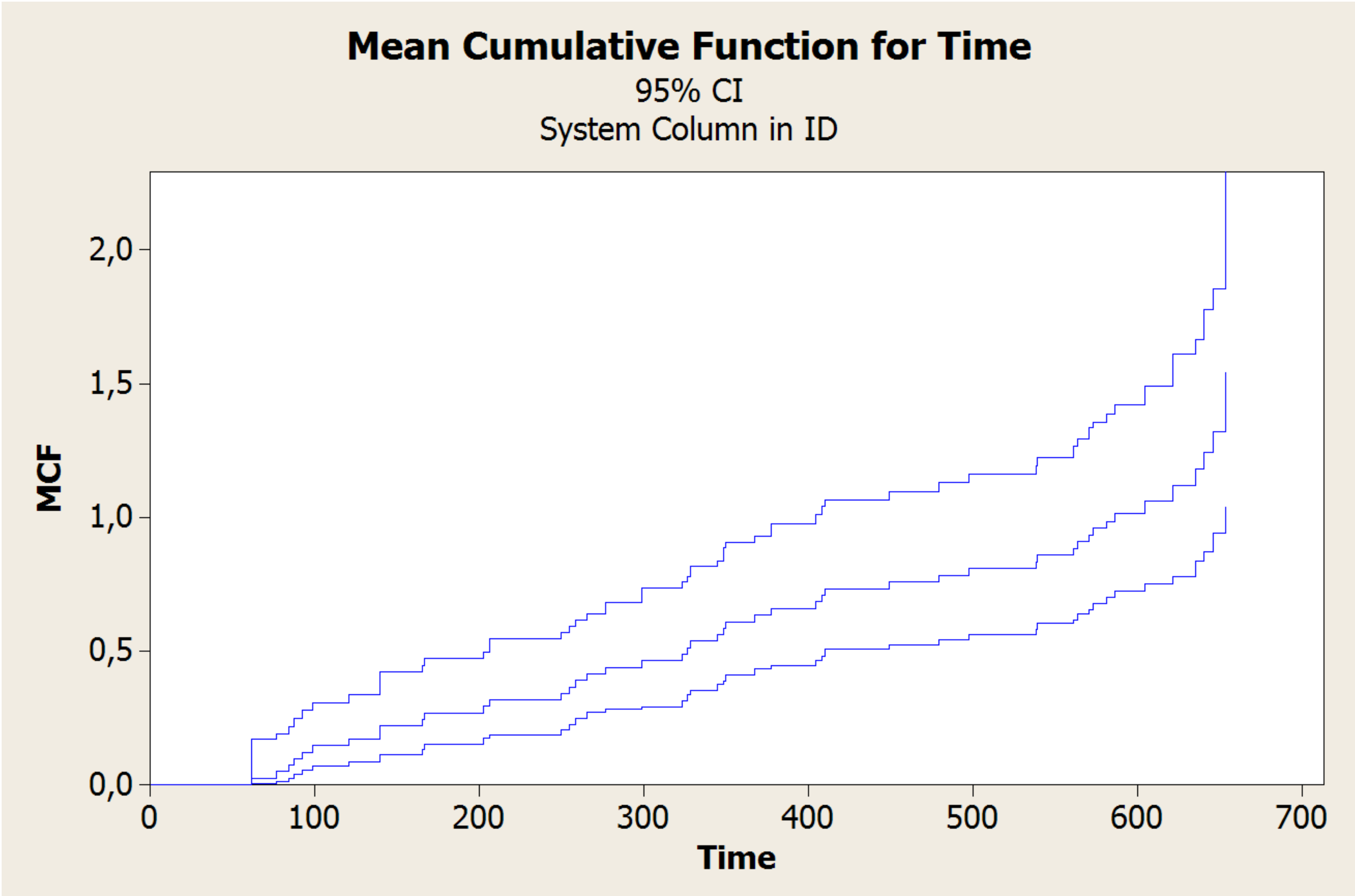


Nonparametric Growth Curve
Data are exact failure/retirement times
Data are interval failure/retirement times
Variables/ Start
Time
End
System Information
System ID
ID
Number of systems:
By variable:
Retirement...
Cost-Freq...
Graphs...
Options...
Storage...
Select
Help
OK
Cancel
Nonparametric Growth Curve - Retirement
Retirement time at largest time for system
Failure truncated systems
Time truncated systems
Retirement time defined by retirement columns
Retirement
Retirement
Select
Help
OK
Cancel

# VALVESEAT DATA



VALVESEAT DATA



# VALVESEAT DATA

## Nonparametric Growth Curve: Time

System: ID

Nonparametric Estimates

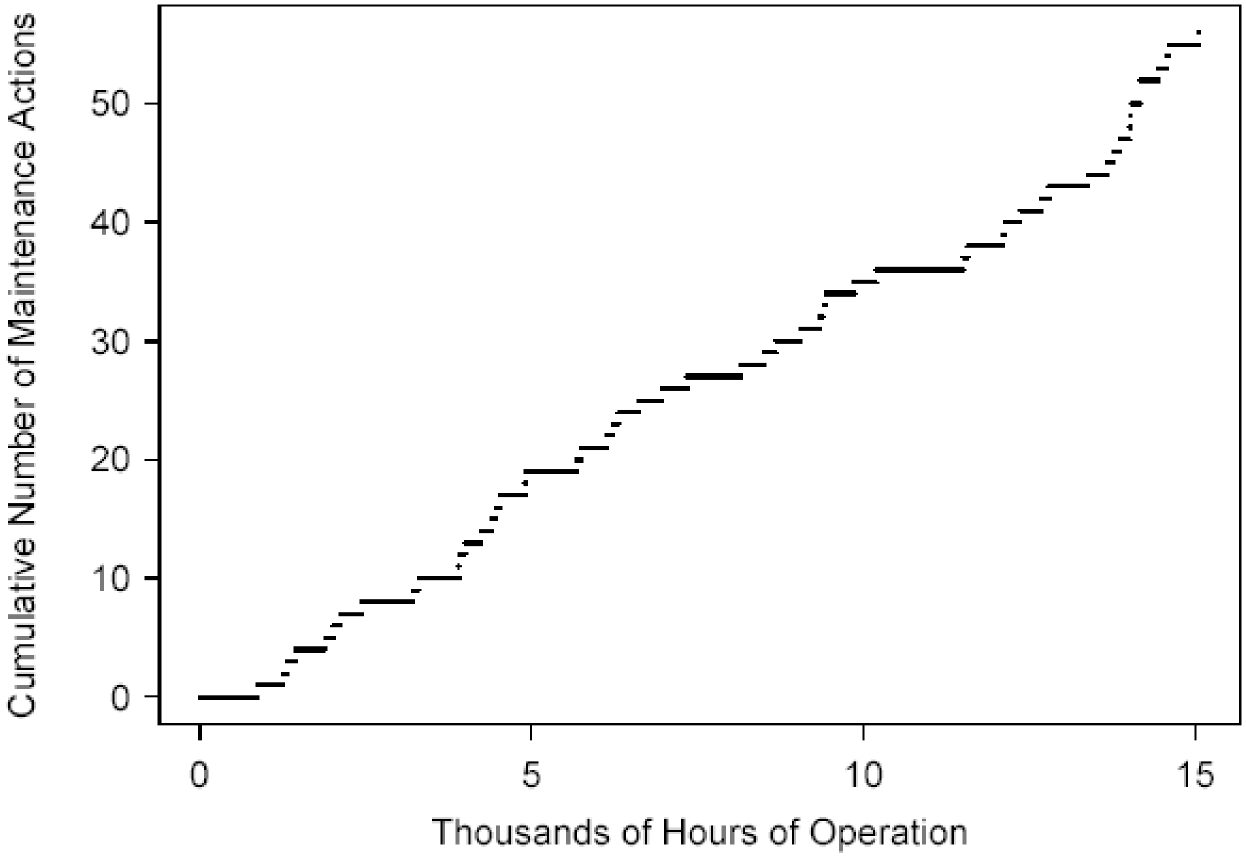
Table of Mean Cumulative Function

| Time | Mean Cumulative Function | Standard Error | 95% Normal CI Lower | 95% Normal CI Upper | System |
|---|---|---|---|---|---|
| 61 | 0,02439 | 0,024091 | 0,00352 | 0,16903 | 12 |
| 76 | 0,04878 | 0,033641 | 0,01262 | 0,18848 | 14 |
| 84 | 0,07317 | 0,040670 | 0,02462 | 0,21750 | 6 |
| 87 | 0,09756 | 0,046340 | 0,03846 | 0,24750 | 7 |
| 92 | 0,12195 | 0,051105 | 0,05364 | 0,27726 | 9 |
| 98 | 0,14634 | 0,055199 | 0,06987 | 0,30650 | 3 |
| 120 | 0,17073 | 0,058764 | 0,08696 | 0,33519 | 19 |
| 139 | 0,19512 | 0,061891 | 0,10479 | 0,36333 | 21 |
| 139 | 0,21951 | 0,073270 | 0,11411 | 0,42226 | 21 |
| 165 | 0,24390 | 0,075417 | 0,13305 | 0,44711 | 24 |
| 166 | 0,26829 | 0,077317 | 0,15251 | 0,47196 | 28 |
| 202 | 0,29268 | 0,078988 | 0,17246 | 0,49672 | 35 |
| 206 | 0,31707 | 0,087527 | 0,18458 | 0,54467 | 28 |
| 249 | 0,34146 | 0,088680 | 0,20525 | 0,56807 | 25 |
| 254 | 0,36585 | 0,089656 | 0,22631 | 0,59143 | 13 |
| 258 | 0,39024 | 0,090461 | 0,24775 | 0,61468 | 11 |
| 265 | 0,41463 | 0,091101 | 0,26955 | 0,63780 | 27 |
| 276 | 0,43902 | 0,097858 | 0,28363 | 0,67955 | 13 |
| 298 | 0,46341 | 0,109607 | 0,29150 | 0,73671 | 13 |
| 323 | 0,48780 | 0,109740 | 0,31387 | 0,75812 | 20 |
| 326 | 0,51220 | 0,109740 | 0,33656 | 0,77949 | 4 |
| 328 | 0,53659 | 0,114907 | 0,35266 | 0,81643 | 11 |
| 344 | 0,56098 | 0,114654 | 0,37581 | 0,83737 | 26 |
| 348 | 0,58537 | 0,124250 | 0,38615 | 0,88737 | 28 |
| 349 | 0,60976 | 0,123782 | 0,40960 | 0,90772 | 16 |
| 367 | 0,63415 | 0,123194 | 0,43334 | 0,92801 | 34 |
| 377 | 0,65854 | 0,131842 | 0,44480 | 0,97498 | 11 |
| 404 | 0,68354 | 0,135939 | 0,46289 | 1,00936 | 16 |

## Times of Unscheduled Maintenance Actions for a USS Grampus Diesel Engine

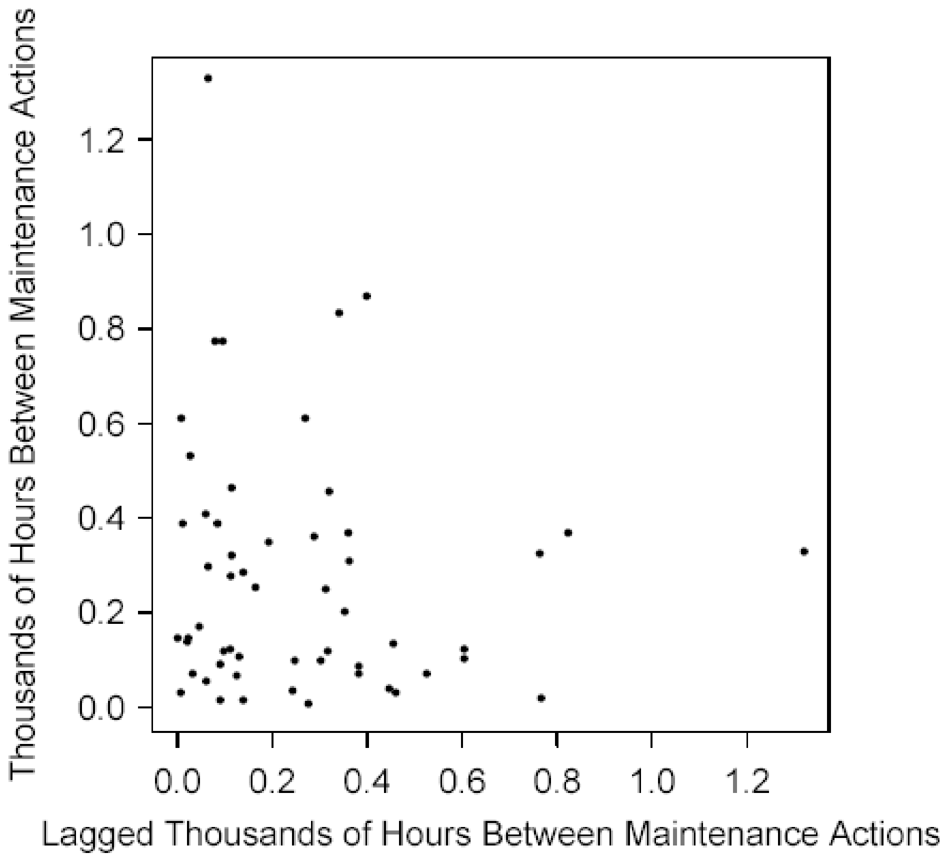
- Unscheduled maintenance actions caused by failure of imminent failure.
- Unscheduled maintenance actions are inconvenient and expensive.
- Data available for 16,000 operating hours.
- Data from Lee (1980).
- Is the system deteriorating (i.e., are failures occurring more rapidly as the system ages)?
- Can the occurrence of unscheduled maintenance actions be modeled by an HPP?

# Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours for a USS Grampus Diesel Engine Lee (1980)



Grampus- data: Plot of $(T_i, T_{i+1})$ to investigate whether times between failures can be assumed independent. The figure does not indicate a correlation between successive times.

**USS Grampus Diesel Engine**
**Plot of Times Between Unscheduled Maintenance Actions Versus Lagged Times Between Unscheduled Maintenance Actions**



## The Likelihood for the NHPP - Single Unit

- With **interval** recurrence data.

  Suppose that the unit has been observed for a period $(0, t_a]$ and the data are the number of recurrences $d_1, \ldots, d_m$ in the nonoverlapping intervals $(t_0, t_1]$, $(t_1, t_2]$, $\ldots$, $(t_{m-1}, t_m]$ (with $t_0 = 0$, $t_m = t_a$).

$$
\begin{aligned}
L(\boldsymbol{\theta}) &= \Pr\left[N(t_0, t_1) = d_1, \ldots, N(t_{m-1}, t_m) = d_m\right] \\
&= \prod_{j=1}^{m} \Pr\left[N(t_{j-1}, t_j) = d_j\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \exp\left[-\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right] \\
&= \prod_{j=1}^{m} \frac{\left[\mu(t_{j-1}, t_j; \boldsymbol{\theta})\right]^{d_j}}{d_j!} \times \exp\left[-\mu(t_0, t_a; \boldsymbol{\theta})\right]
\end{aligned}
$$

## The Likelihood for the NHPP (Continued)

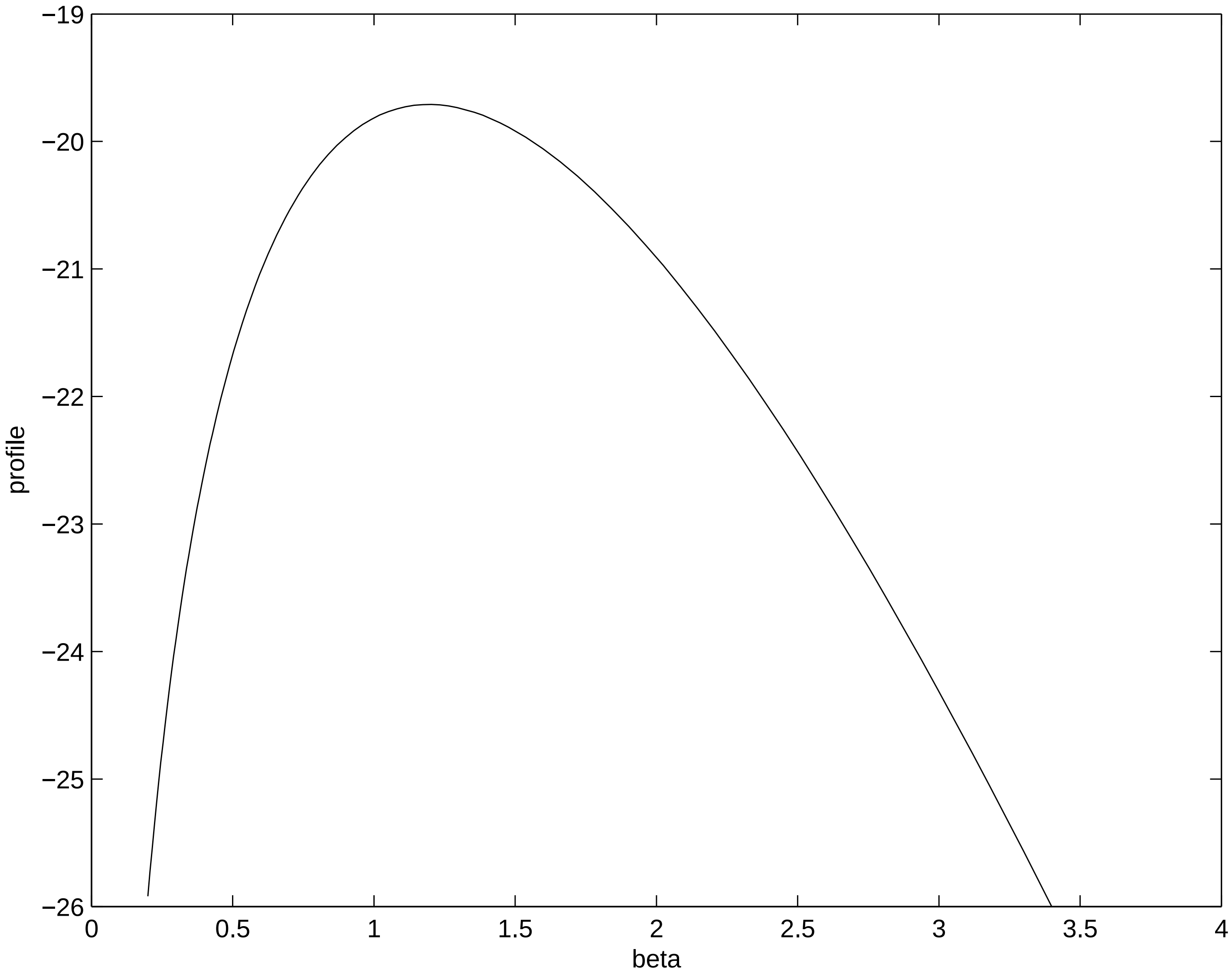
- If the number of intervals $m$ increases and there are **exact** recurrences at $t_1 \leq \ldots \leq t_r$ (here $r = \sum_{j=1}^{m} d_j$, $t_0 \leq t_1$, $t_r \leq t_a$), then using a limiting argument it follows that the likelihood in terms of the density approximation is

$$L(\boldsymbol{\theta}) = \prod_{j=1}^{r} \nu(t_j; \boldsymbol{\theta}) \times \exp\left[-\mu(0, t_a; \boldsymbol{\theta})\right]$$

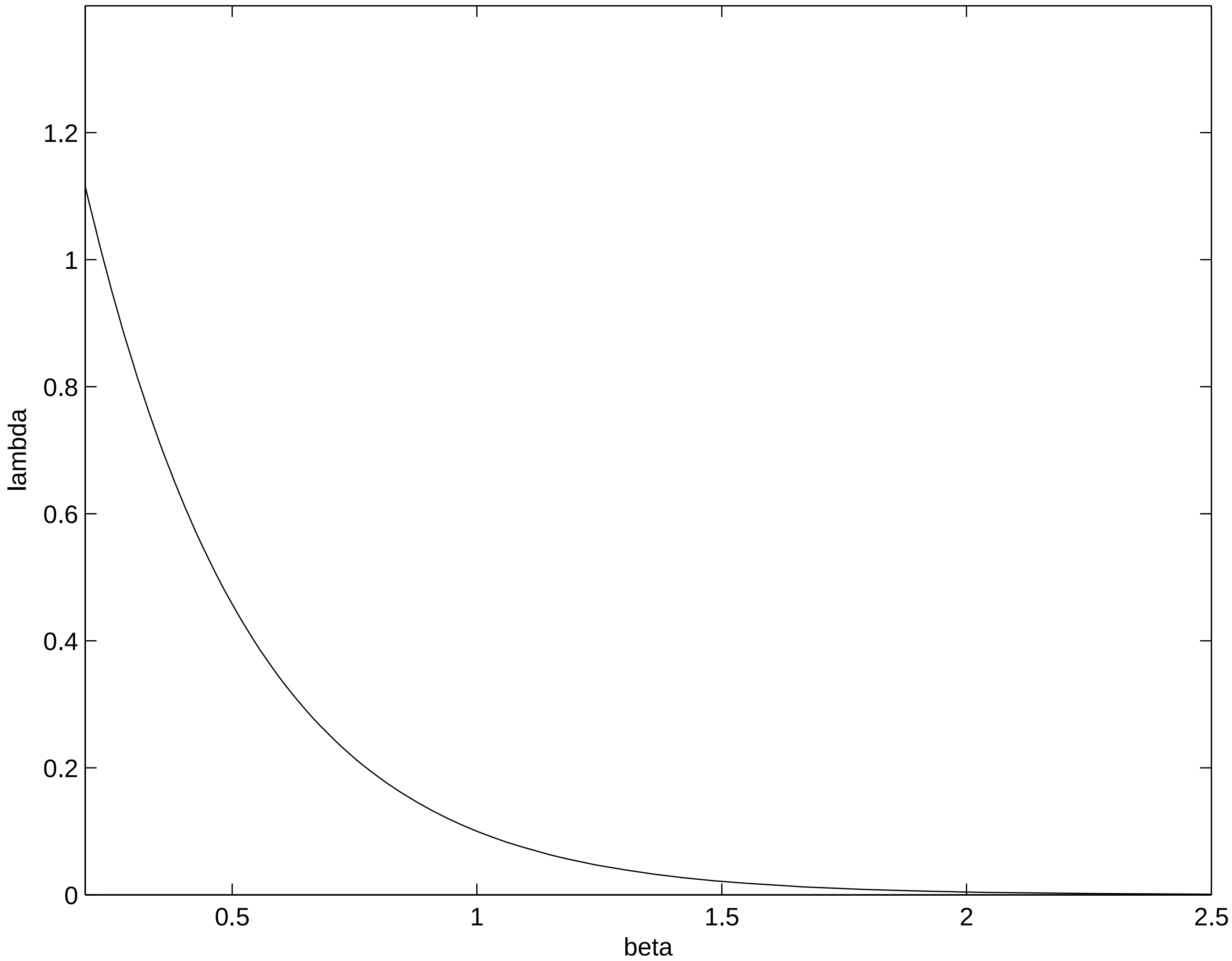
- For simplicity, above we assumed that the intervals are contiguous. Obvious changes to the formula above give the likelihood when there are gaps among the intervals.

- In both cases (the interval data or exact recurrences data) the same methods used in Chapters 7, 8 can be used to obtain the ML estimate $\widehat{\boldsymbol{\theta}}$ and confidence regions for $\boldsymbol{\theta}$ or functions of $\boldsymbol{\theta}$.

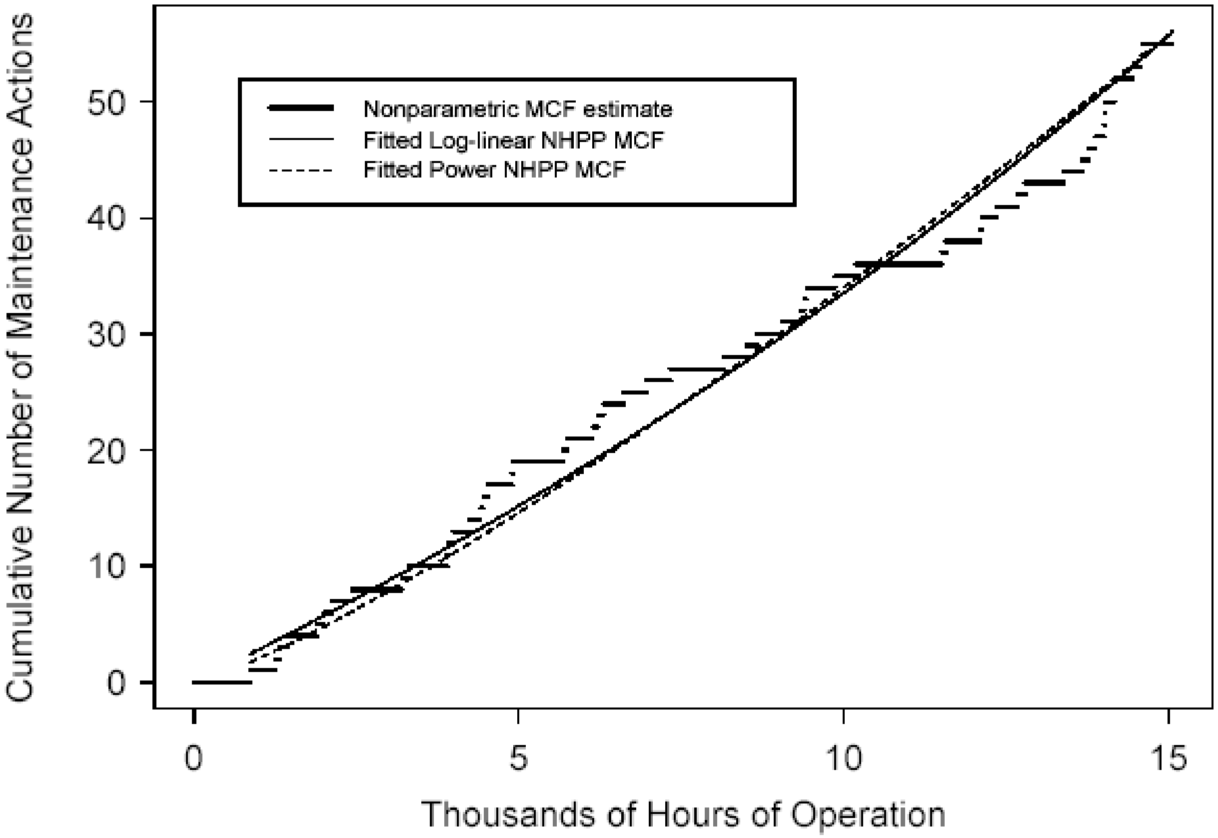
PROFILE LIKELIHOOD FOR BETA
("SIMPLE EXAMPLE")
$\widehat{\beta} = 1.20,\ \widehat{\lambda} = 0.0538.$



CONNECTION BETWEEN LAMBDA OG BETA
("SIMPLE EXAMPLE")
$\widehat{\beta} = 1.20,\ \widehat{\lambda} = 0.0538.$



# Cumulative Number of Unscheduled Maintenance Actions Versus Operating Hours with Power and Loglinear NHPP Models for a USS Grampus Diesel Engine



## Results of Fitting NHPP Models to the USS Grampus Diesel Engine Data

- Both models seem to fit the data very well.

- For the power recurrence rate model, $\widehat{\beta}$=1.22 and $\widehat{\eta}$ =0.553.

- For the loglinear recurrence rate model, $\widehat{\gamma}_0$=1.01 and $\widehat{\gamma}_1$ =.0377.

- Times between recurrences are consistent with a HPP:

  - the Lewis-Robinson test gave $Z_{\text{LR}} = 1.02$ with $p$-value $p = .21$.

  - the MIL-HDBk-189 test gave $X^2_{\text{MHB}} = 92$ with $p$-value $p = .08$.

## Comparison of trend tests

main topic

Minitab provides five trend tests for data with multiple systems: MIL-hdbk-189 (TTT-based), MIL-hdbk-189 (Pooled), Laplace's (TTT-based), Laplace's (Pooled), and Anderson-Darling. The pooled Laplace and military handbook tests reduce to their respective TTT-based tests when there is only one system. These tests behave differently under the following two circumstances:

1 the data follow a non-monotonic trend

2 the data are from heterogeneous systems

### Monotonic and non-monotonic trends

There is a trend in the pattern of times between failure if the times change in a systematic way. Trends can be:

- monotonic – times between failures are getting either consistently longer (decreasing trend) or consistently shorter (increasing trend)
- non-monotonic – times between failures alternate between increasing and decreasing trend (cyclic) or have a decreasing trend, no trend, and then increasing trend (bathtub)

The Anderson-Darling test will reject the null hypothesis in the presence of both monotonic and non-monotonic trends. The other tests will generally only detect monotonic trends. While the Anderson-Darling test is useful if you suspect the existence of a cyclic or other non-monotonic trend, the other tests are more powerful in the case of a monotonic trend.

### Homogeneous and heterogeneous systems

The null hypothesis of no trend differs slightly for the different tests:

- The null hypothesis for the pooled tests (MIL-hdbk-189 and Laplace's) is that the data come from a homogeneous Poisson processes (HPP) with a possibly different MTBF for each system. Thus, rejecting the null hypothesis means that you can definitely conclude there is a trend in your data.
- The null hypothesis for the TTT-based tests (MIL-hdbk-189, Laplace's, and Anderson-Darling) is that the data come from a homogeneous Posson process (HPP) with the same MTBF for each system. Thus, rejecting the null hypothesis could mean that either there is a trend in your data or your data come from heterogeneous systems. Therefore, you should use TTT-based tests only when you are confident that your systems are homogeneous.

The table below summarizes the different null hypotheses associated with the trend tests.

| | MIL-hdbk-189 (Pooled) | MIL-hdbk-189 (TTT-based) | Laplace's (Pooled) | Laplace's (TTT-based) | Anderson-Darling |
|---|---|---|---|---|---|
| **Null Hypothesis** | HPP (possibly different MTBFs) | HPP (equal MTBFs) | HPP (possibly different MTBFs) | HPP (equal MTBFs) | HPP (possibly different MTBFs) |
| **Rejecting $H_0$ means...** | monotonic trend | monotonic trend or systems are heterogeneous | monotonic trend | monotonic trend or systems are heterogeneous | monotonic trend or non-monotonic trend or systems are heterogeneous |

See [12] for more information concerning these tests.

Reliability Engineering and System Safety 60 (1998) 13-28

0951-8320/98/$19.00

PII: S0951-8320(97)00099-9

# TTT-based tests for trend in repairable systems data

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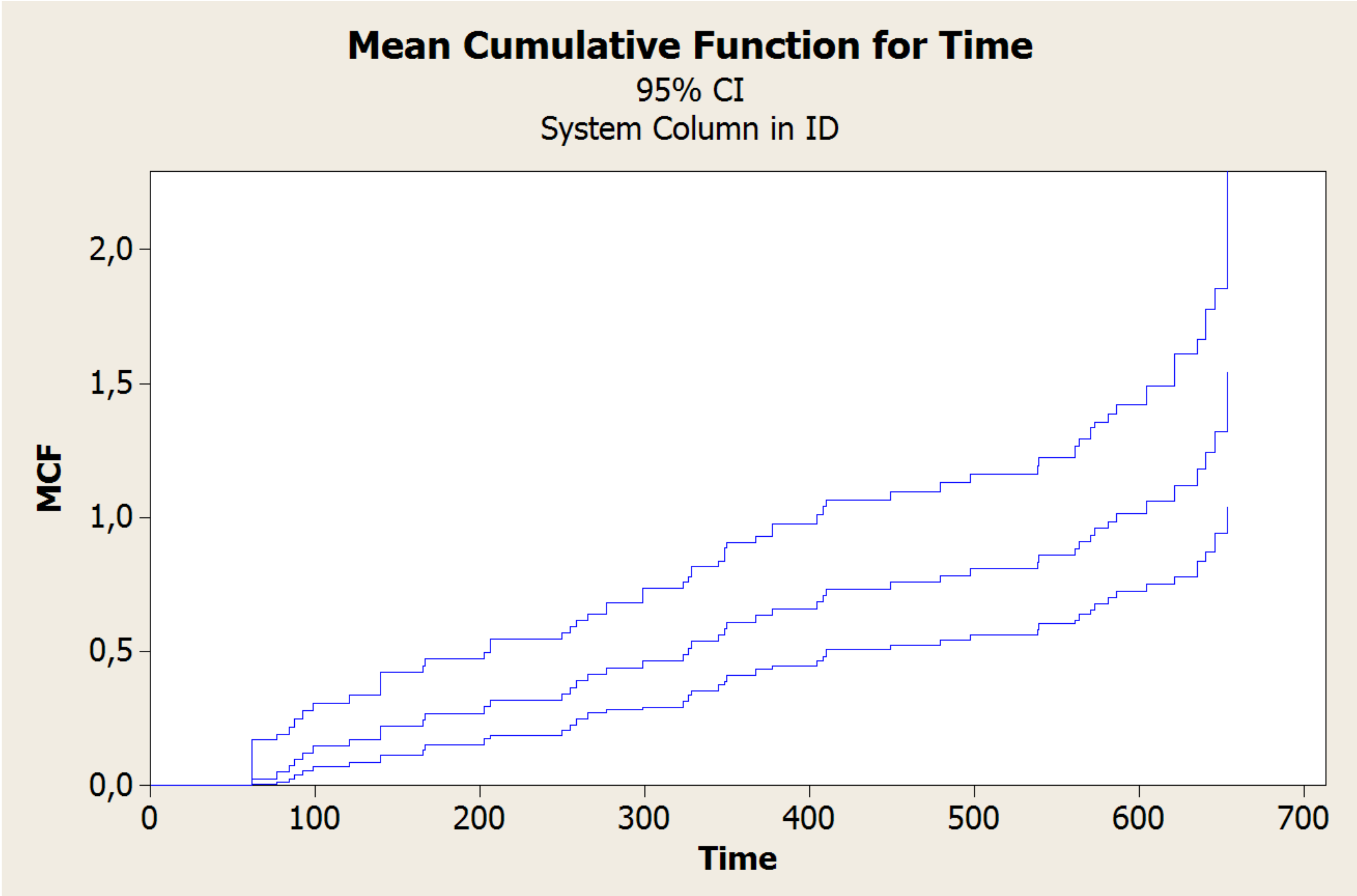
A major aspect of analysis of failure data for repairable systems is the testing for a possible trend in interfailure times. This paper reviews some important and popular graphical methods and tests for the nonhomogeneous Poisson process model. In particular, the total time on test (TTT) plot is considered, and trend tests based on the TTT-statistic are motivated and derived. In particular, a test based on the Anderson-Darling statistic is suggested. The tests are evaluated and compared in a simulation study, both with respect to the achievement of correct significance level and rejection power. The considered alternatives to 'no trend' are the log-linear, power law and a class of bathtub-shaped intensity functions. The simulation study involves single systems, as well as the case where several independent systems of the same kind are observed. © 1998 Elsevier Science Limited.

Valveseat Data



## Valveseat Data

Trend Tests

| | MIL-Hdbk-189 | | Laplace's | | |
|---|---|---|---|---|---|
| | TTT-based | Pooled | TTT-based | Pooled | Anderson-Darling |
| Test Statistic | 80,28 | 66,15 | 0,46 | 2,38 | 0,80 |
| P-Value | 0,249 | 0,017 | 0,645 | 0,017 | 0,478 |
| DF | 96 | 96 | | | |

## TTT-analysis Simple Example

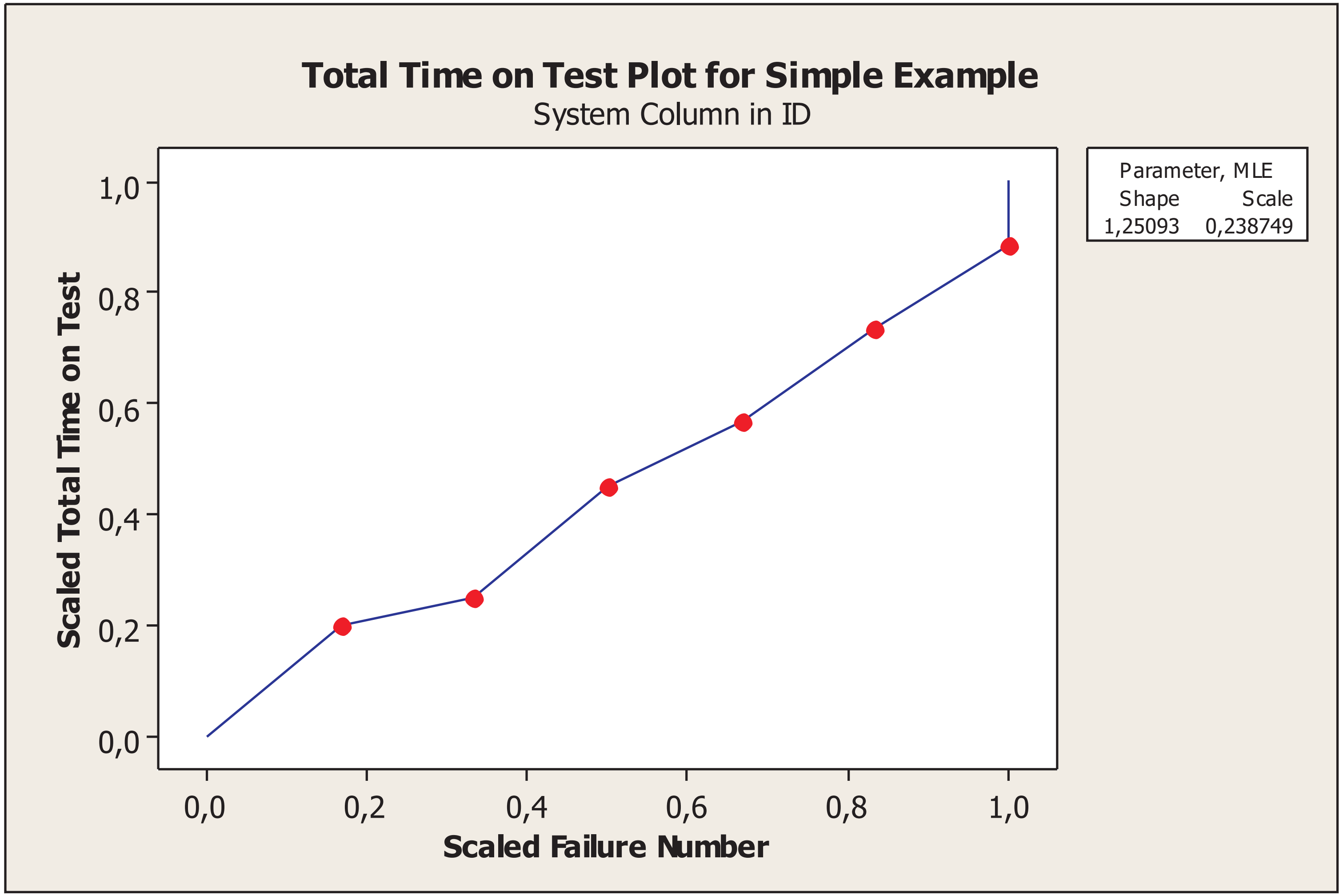
| Row | STTT | ID | Scaled |
|---|---|---|---|
| 1 | 12 | 1 | 0,20000 |
| 2 | 15 | 1 | 0,25000 |
| 3 | 27 | 1 | 0,45000 |
| 4 | 34 | 1 | 0,56667 |
| 5 | 44 | 1 | 0,73333 |
| 6 | 53 | 1 | 0,88333 |
| 7 | 60 | 1 | 1,00000 |

Parameter Estimates

| | | Standard | 95% Normal CI | |
|---|---|---|---|---|
| Parameter | Estimate | Error | Lower | Upper |
| Shape | 1,25093 | 0,511 | 0,249996 | 2,25186 |
| Scale | 0,238749 | 0,160 | -0,0746105 | 0,552109 |

Trend Tests

| | MIL-Hdbk-189 | Laplace's | Anderson-Darling |
|---|---|---|---|
| Test Statistic | 9,59 | 0,12 | 0,24 |
| P-Value | 0,697 | 0,906 | 0,977 |
| DF | 12 | | |


Total Time on Test Plot for Simple Example
System Column in ID
Parameter, MLE
Shape Scale
1,25093 0,238749
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
Scaled Total Time on Test
Scaled Failure Number

## TTT-analysis of Valve Seat Data

Parametric Growth Curve: C1

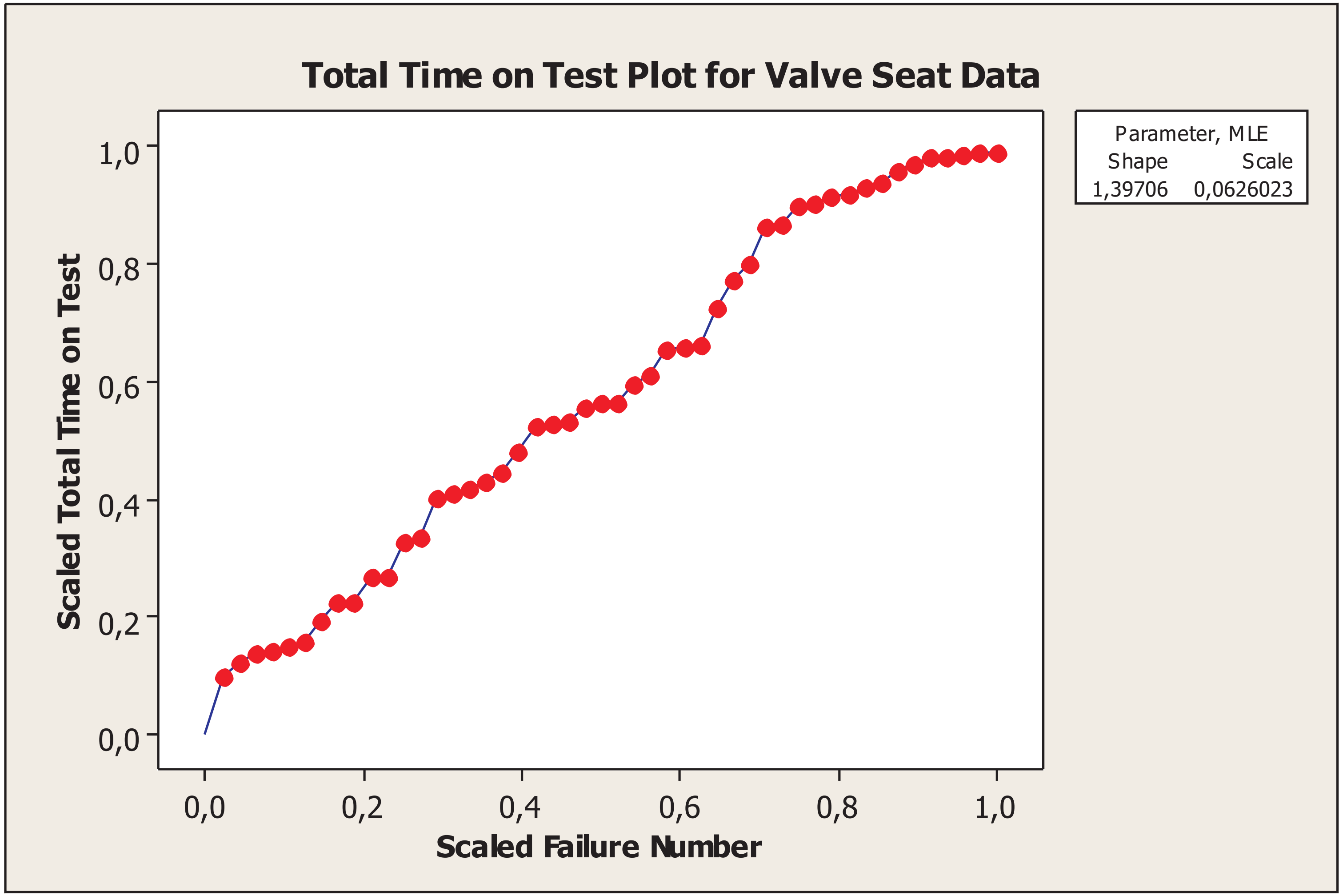
Model: Power-Law Process
Estimation Method: Maximum Likelihood

Parameter Estimates

| | | Standard | 95% Normal CI | |
|---|---|---|---|---|
| Parameter | Estimate | Error | Lower | Upper |
| Shape | 1,39706 | 0,202 | 1,00184 | 1,79229 |
| Scale | 0,0626023 | 0,026 | 0,0119179 | 0,113287 |

Trend Tests

| | MIL-Hdbk-189 | Laplace's | Anderson-Darling |
|---|---|---|---|
| Test Statistic | 68,72 | 2,03 | 3,17 |
| P-Value | 0,032 | 0,043 | 0,022 |
| DF | 96 | | |



Total Time on Test Plot for Valve Seat Data
Scaled Total Time on Test
1,0
0,8
0,6
0,4
0,2
0,0
0,0
0,2
0,4
0,6
0,8
1,0
Scaled Failure Number
Parameter, MLE
Shape
Scale
1,39706
0,0626023