

TMA4275 Lifetime Analysis (Spring 2015)
Exercise 6

Problem 1 – The two-parameter exponential distribution

The two-parameter exponential distribution has density

$$f(t; \theta, \gamma) = \frac{1}{\theta} \exp \left\{ -\frac{t - \gamma}{\theta} \right\} \text{ for } t \geq \gamma$$

where $\theta > 0$, $\gamma \geq 0$.

Assume that we have a right censored sample (y_i, δ_i) , $i = 1, \dots, n$ from this distribution. Assume that all censorings take place at or after time γ .

- a) Find the log-likelihood function $l(\theta, \gamma)$ for these data.
- b) Let $(\hat{\theta}, \hat{\gamma})$ be the maximum likelihood estimators of (θ, γ) . Why do we have

$$\hat{\gamma} \leq y_{(1)}$$

where $y_{(1)}$ is the smallest time among y_1, \dots, y_n ?

Then find explicit expressions for $(\hat{\theta}, \hat{\gamma})$. Show in particular that we always have $\hat{\gamma} = y_{(1)}$

- c) In the lectures we have considered a likelihood method for constructing confidence intervals for one of two parameters in a model. The method uses the following:

Let $\hat{\gamma}(\theta)$ be the MLE of γ when θ is given. Then

$$W(\theta) = 2(\ell(\hat{\theta}, \hat{\gamma}) - \ell(\theta, \hat{\gamma}(\theta)))$$

is approximately χ_1^2 when θ is the true parameter. (Note that $\tilde{\ell}(\theta) = \ell(\theta, \hat{\gamma}(\theta))$ is the so-called profile log likelihood of θ .)

Explain how this can be used to construct a confidence interval for θ . Do the calculations of the interval as far as you get.

- d) Use MINITAB to estimate the parameters when the Pike cancer data (see page 25 of Slides 10 from lectures) are assumed to follow a two-parameter exponential distribution.
- e) Reconsider the assumption in the beginning of the Problem, that all censorings take place at or after time γ . Can you think of cases where censorings also before time γ are possible? In such cases, how should the analysis in b) be modified?

Problem 2 – Censoring and truncation

$n = 10$ units with exponentially distributed life times and $\text{MTTF} = \theta$ are put on test. At time $c = 10$ the test is ended (type I censoring), and $r = 4$ units have failed by that time. The observed lifetimes are

0.9, 2.8, 5.9, 7.4

- a) Write down the likelihood function and compute the MLE for θ . Which are the assumptions behind this approach?
- b) Assume now that at the end of the experiment ($c = 10$) one does not know how many units were put on test, but only knows that the experiment has gone for 10 time units, with $r = 4$ failures at the times given.
How can you write down a likelihood for this case? (Hint: This is right truncation, see page 3 of Slides 11 from lectures).
Which are the assumptions behind this likelihood?
- c) Maximize the likelihood in (b) to find the MLE under the conditions given there.

Problem 3 – Weibull regression

Do Problem 1 b,c in Exam 2013V.