Norges teknisk–
naturvitenskapelige universitet
Institutt for matematiske fag

Contact during exam:
Jayant Deshpande (735) 93520

EXAM IN TMA4275 LIFETIME ANALYSIS

Wednesday 6 June 2007

Time: 09:00–13:00

Aids:
All printed and handwritten sources. Particular, simple calculator.

Grading: 27 June 2007

ENGLISH

Problem 1

a) Explain what is meant by (i) time censored, (ii) order censored, and (iii) random (right) censored observations. How do they arise in practice?

b) Show that the hazard rate of a series system of independent components is equal to the sum of the components' hazard rates.

c) Explain the independent latent lifetimes model in competing risks and its limitations.

d) Let $p_1 < p_2 < \ldots < p_k$ be the reliabilities of k independent components which form a series system. If $I_1, I_2, \ldots, I_k$ are the corresponding (Birnbaum) importance measures of these components, then show that

$$I_1 > I_2 > \ldots > I_k.$$

Problem 2

Let the failure times of a repairable system be governed by an NHPP with loglinear intensity:

$$\lambda(t) = e^{\alpha+\beta t}, \quad 0 < \alpha, \beta < \infty, \quad 0 \leq t.$$

a) For what values of β is the system improving or deteriorating?

b) What is the survival function of the time to first failure?

c) Suppose that the failure process is observed until the nth failure. Assuming that t_n, the last failure time, is given, set up the likelihood equation for obtaining the maximum likelihood estimates of α and β on the basis of the data $0 < t_1 < t_2 < \ldots < t_n$.

Problem 3

The following are the mileages at which 19 military personnel carriers failed in service. There were no censored observations.

162	200	271	320	393	508	539
629	706	777	884	1008	1101	1182
1463	1603	1484	2355	2880		

a) Plot the scaled TTT transform for the above data and comment on the suitability of the exponential model for it.

b) Carry out a test of exponentiality against the IFRA alternative at 5% level of significance.

c) Assuming exponential distribution, obtain a 90% confidence interval for θ, its mean.

Problem 4

Below are given the survival times of 14 advanced lung cancer patients. These were randomly designed to a "standard group" and a "test group".

Standard	411	126	118	82	8	25*	11
Test	999	231*	991	1	201	44	15

* denotes censored observation

a) Obtain the Kaplan-Meier estimators of the two survival functions and plot them in the same graph.

b) Carry out the Mantel-Haenzel test for the null hypothesis that the two random samples are from the same survival function.

Problem 5

a) Describe the proportional hazards model with a loglinear link function and the corresponding partial likelihood function.

In an experiment the lifetime is influenced by two covariates, $Z_1 = 0, 1$ (according to whether treatment or placebo is used) and Z_2 (continuous) denoting the wellness measure assigned by the physician. The maximum partial likelihood estimates of the regression coefficients and their estimated variances under the above model are

Covariate	$\hat{\beta}$	$\widehat{\text{Var}}(\hat{\beta})$
Z_1	-0.383	0.0169
Z_2	-0.0206	0.0000784

b) Separately test $\text{H}_0 : \ \beta_1 = 0$ and $\text{H}_0' : \ \beta_2 = 0$ using Wald's test.

c) Also, if the baseline hazard rate is given by $h_0(t) = \frac{5}{2}\, t^2$, then obtain the hazard rates for

$$\begin{aligned} & Z_1 = 1, \quad Z_2 = -10 \\ \text{and} \quad & Z_1 = 1, \quad Z_2 = -5. \end{aligned}$$

and plot them together with the baseline hazard rate on the same graph for $0 < t < 200$.

d) Comment on the effect of Z_2 as shown by these results.