TMA 4295 Statistical Inference 2014
Homework 4

Problem 1

Let $X_1, \dots, X_n$ be i.i.d. uniformly distributed on the interval $[0, \theta]$.

a) Prove that the statistic

$$T(\boldsymbol{X}) = \max\{X_1, \dots, X_n\}$$

is sufficient for θ.

b) Find a sufficient statistic also when $X_1, \dots, X_n$ are uniformly distributed on the interval $[-\theta, \theta]$.

Problem 2

Let $X_1, \dots, X_n$ be i.i.d. Bernoulli-distributed with parameter p. Let

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Use Theorem 6.2.13 to prove that $\bar{X}_n$ is minimal sufficient.

Problem 3

Do Exercise 6.9 (b).

Hint: Prove that $T(\boldsymbol{X}) = \min(X_1, \dots, X_n)$ is minimal sufficient.

Problem 4

Let $X_1, \dots, X_n$ be i.i.d. from some distribution with density $f(x|\theta)$.

a) Have a look at the definition of order statistics in Def. 5.4.1. Then show that the order statistics is sufficient for θ, without the need for further specification of $f(x|\theta)$.

Hint: The joint density of the order statistics is found on page 230 in the book (8 lines from the bottom of the page). Verify the expression! Then use the factorization theorem to prove sufficiency.

b) Let now $f(x|\theta)$ be given as in Exercise 6.9 (c). Show that the order statistic is minimal sufficient in this case.

Problem 5

Let $\boldsymbol{X} = (X_1, \ldots, X_n)$ be i.i.d. observations of a random variable X with density

$$f(x|\theta) = \theta(\theta+1)x^{\theta-1}(1-x) \text{ for } 0 < x < 1$$

where $\theta > 0$ is an unknown parameter.

a) Show that the distribution of X can be written as a (one-parameter) exponential family (see p. 111 in book).

Then find a sufficient statistic for θ based on $\boldsymbol{X}$ (see, e.g., Theorem 6.2.10).

b) Show that for a single observation,

$$E_\theta(X) = \frac{\theta}{\theta+2}$$

Use this to find a moment estimator for θ.

c) Find the MLE of θ.

d) Which objection would you have to the moment estimator?