THE CAMP CHIEF AND "N"
(A LARGE INTERNATIONAL FOOD PROCESSING COMPANY)

4180 Optimeringsteori
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• The Camp Chief is responsible for feeding the participants

• She buys $N$ different types of food $\{X_i\}_{i=1}^{N}$ (apples, bread, milk, ...)

• Prices per unit: $C_i$

• Cost of feeding one person one day:

$$\sum_{i=1}^{N} C_i x_i$$

The Camp Chief has a tight budget and wants to feed the camp at lowest possible price!
Each type of food contains a certain fraction of $M$ basic ingredients (fat, carbohydrates, vitamins, minerals,...):

$$A = \left\{ a_{ji} \right\}$$

(basic ingredient ”$j$”, $i$ = food no. ”$i$”)

The National Health Organization:

*The camp must have a balanced daily diet:*

$$\sum_{i=1}^{N} a_{ji}x_i \geq b_j, \ j = 1, \ldots, M$$
The Chief’s optimization problem:

\[ \text{min} \sum_{i=1}^{N} c_i x_i \]

\[ \sum_{i=1}^{N} a_{ji} x_i \geq b_j, \quad j = 1, \ldots, M \]

\[ x_i \geq 0, \quad i = 1, \ldots, N \]
Now N enters the scene:

N produces **pure** basic ingredients and proposes that the Chief should buy those and

\[
\textit{synthesize the food!}
\]

N’s problem: How to price the basic ingredients so as to maximize their own profit:

Unit prices: \( \lambda_i, i = 1, \ldots, M \)

Price for one unit of \( X_i \):

\[
\lambda_1 a_{1i} + \lambda_2 a_{2i} + \cdots + \lambda_M a_{Mi}
\]
Recall NHO: *Daily requirement at the camp of basic ingredient* $j$:

$$b_j$$

**N’s optimization problem:**

*maximum profit, but acceptable price:*

$$\max \left\{ \sum_{j=1}^{M} \lambda_j b_j \right\}$$

$$\lambda_1 a_{1i} + \lambda_2 a_{2i} + \cdots + \lambda_m a_{mi} \leq c_i, \ i = 1, \ldots, N$$

$$\lambda \geq 0$$
THE PRIMAL PROBLEM
(The Chief)

\[ \begin{align*}
\min & \quad c'x \\
Ax & \geq b, \\
x & \geq 0
\end{align*} \]

THE DUAL PROBLEM
(N)

\[ \begin{align*}
\max & \quad \lambda'b \\
\lambda' A & \leq c, \\
\lambda & \geq 0
\end{align*} \]

The Duality Theorem