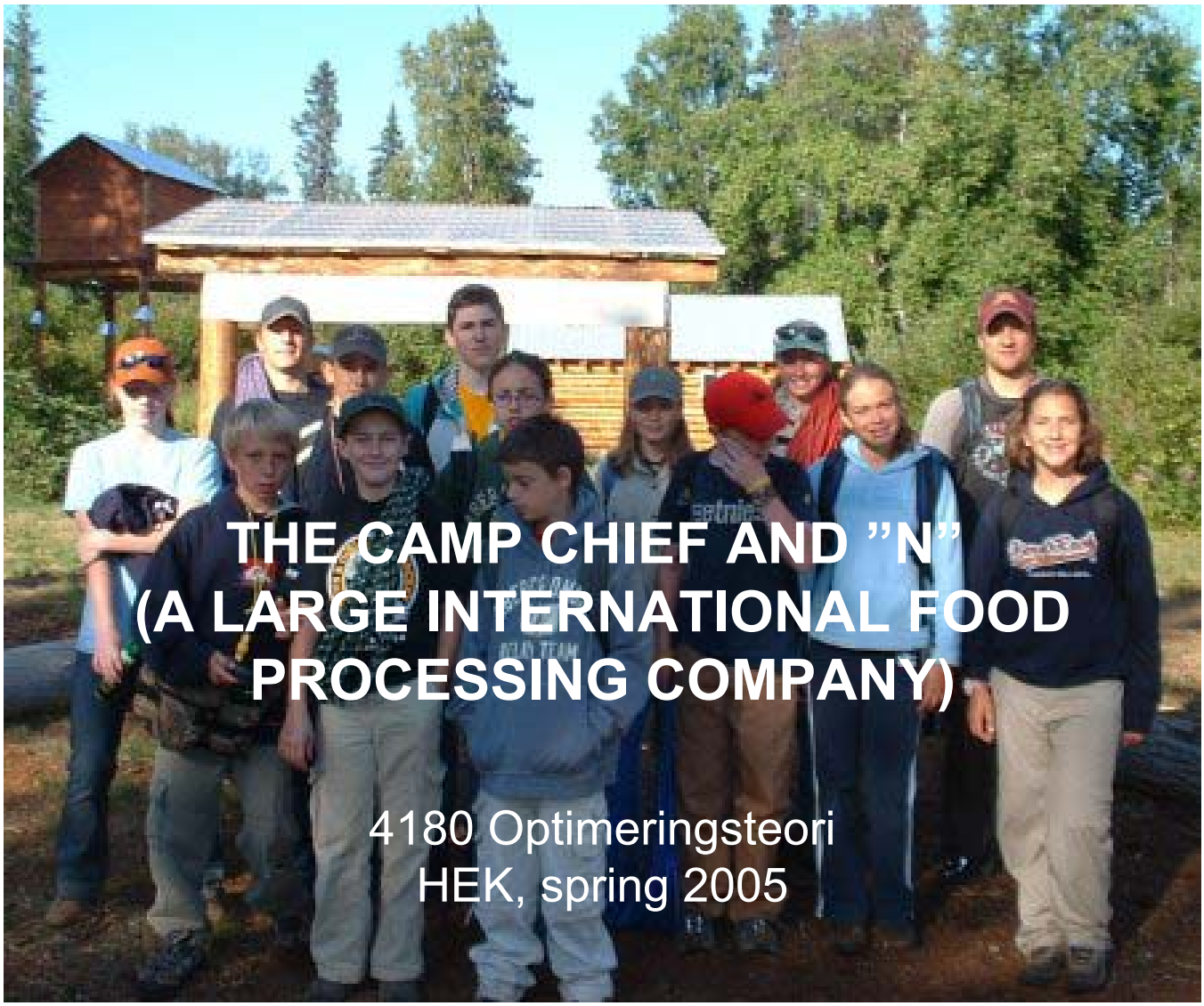


THE CAMP CHIEF AND ”N” (A LARGE INTERNATIONAL FOOD PROCESSING COMPANY)

4180 Optimeringsteori
HEK, spring 2005

- The Camp Chief is responsible for feeding the participants
- She buys N different types of food $\{X_i\}_{i=1}^{N}$ (apples, bread, milk ,...)
- Prices per unit: c_i
- Cost of feeding one person one day:

$$\sum_{i=1}^{N} c_i x_i$$

The Camp Chief has a tight budget and wants to feed the camp at lowest possible price!

Each type of food contains

a certain fraction of M basic ingredients

(fat, carbohydrates, vitamins, minerals,...):

$$A = \{a_{ji}\}$$

(basic ingredient "j" , i = food no. "i")

The National Health Organization:
The camp must have a balanced daily diet:

$$\sum_{i=1}^{N} a_{ji} x_i \geq b_j, \; j = 1,\ldots,M$$

The Chief's optimization problem:

$$\min_x \sum_{i=1}^{N} c_i x_i$$

$$\sum_{i=1}^{N} a_{ji} x_i \geq b_j,\ j = 1,\ldots,M$$

$$x_i \geq 0, i = 1,\ldots,N$$

Now N enters the scene:

N produces **pure** basic ingredients and proposes that the Chief should buy those and

synthesize the food!

N's problem: How to price the basic ingredients so as to maximize their own profit:

Unit prices: $\lambda_i, i = 1,...,M$

Price for one unit of X_i :

$$\lambda_1 a_{1i} + \lambda_2 a_{2i} + \cdots + \lambda_M a_{Mi}$$

Recall NHO: *Daily requirement at the camp of basic ingredient* j:

$$b_j$$

N's optimization problem:
maximum profit, but acceptable price:

$$\max\left\{\sum_{j=1}^{M}\lambda_j b_j\right\}$$

$$\lambda_1 a_{1i} + \lambda_2 a_{2i} + \cdots + \lambda_m a_{mi} \le c_i,\ i = 1,\ldots,N$$

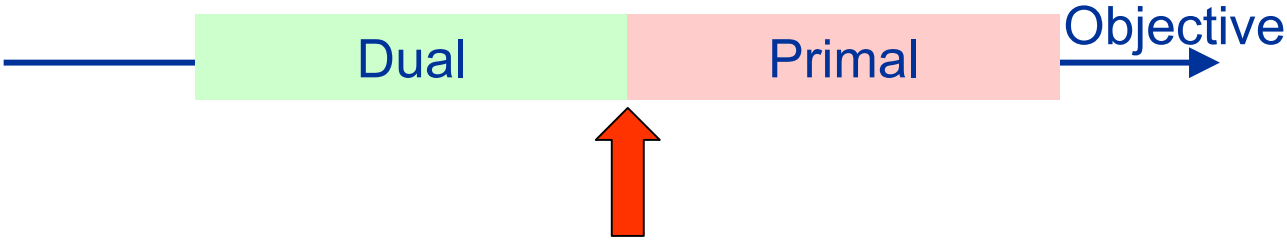
$$\lambda \ge 0$$

THE PRIMAL PROBLEM
(The Chief)

$$\min c'x$$
$$Ax \geq b,$$
$$x \geq 0$$

THE DUAL PROBLEM
(N)

$$\max \lambda' b$$
$$\lambda' A \leq c,$$
$$\lambda \geq 0$$



The Duality Theorem