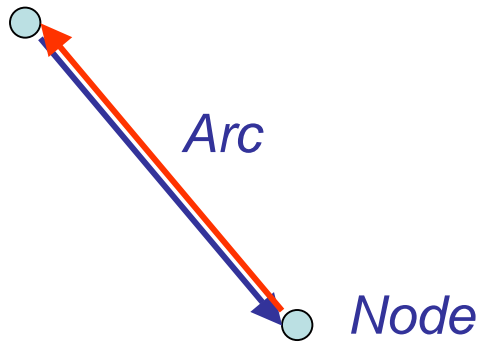


TMA 4180 Optimeringsteori

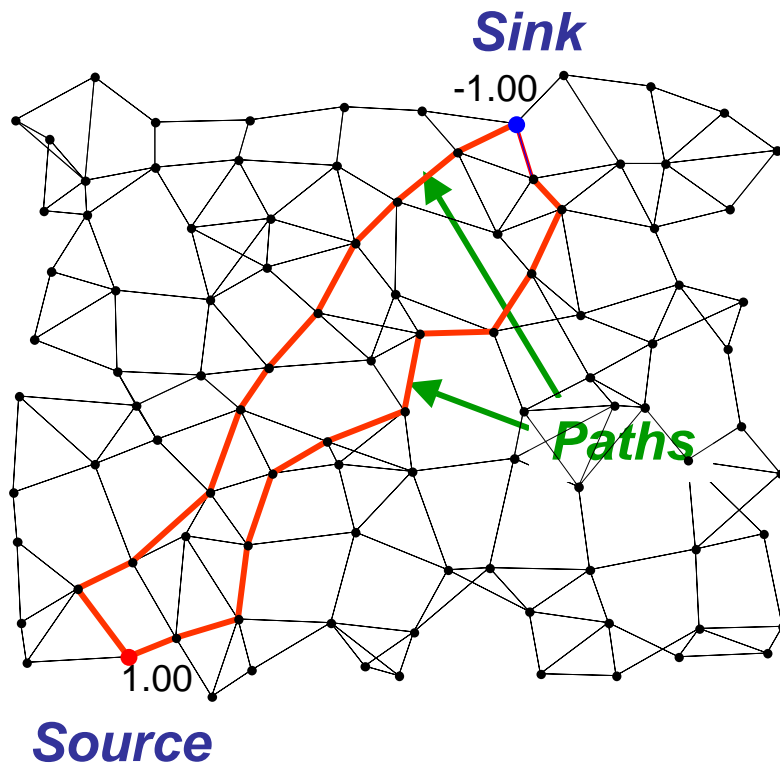
Minimum Cost Network Flow
Analysis Using LP

Harald E. Krogstad
March 2007



An arc is characterized by

- Prize pr. flow unit along arc
- Lower bound (for initiating transport)
- Upper bound (capacity)



Sources: (Production/providers)

- Capacity
- Cost pr. unit delivered to the network

Sinks (Consumers/receivers):

- Capacity
- *Income* to network from deliveries

Source: Production $b > 0$.

Sink: Absorption, $b < 0$.

Variables $x = \{x_i\}, x_i \geq 0$. (flow in the arcs)

NB! 2 variables for each arc: 2 *directions*

Node:
$$\sum_{\text{inflow}} x_i = \sum_{\text{outflow}} x_i$$

Source/Sink:
$$b_s = \sum_{\text{outflow}} x_i - \sum_{\text{inflow}} x_i$$

A *balanced* network:
$$\sum_{\text{Sources/sinks}} b_s = 0$$

Price for delivery:
$$f(x) = \sum_{\text{arcs}} c_i x_i = c'x$$

Cost for one unit along arc “ i ”: $\{c_i\}$

Upper bound on capacity for arc “ i ”: $\{ub_i\}$

Lower bound on capacity for arc “ i ”: $\{lb_i\}$

The LP formulation:

$$\begin{aligned} & \min_x c'x \\ & \sum_{\text{outflow}} x_i - \sum_{\text{inflow}} x_i = b_n, n = 1, \dots, \text{Nodes}, \\ & lb \leq x \leq ub. \end{aligned}$$

$$\begin{aligned} & \min_x c'x \\ & A_{eq}x = b_{eq} \\ & lb \leq x \leq ub \end{aligned}$$

The matrix is a *sparse* matrix with only -1, 0, and 1

Simsys_sparse



An open exchange for the MATLAB and Simulink user community

<http://www.mathworks.com/matlabcentral/>

Per Bergström
Luleå University of Technology



RANDOM NETWORK GENERATION

Prescribe:

- Numbers of sources and sinks
- Max number of arcs around one node
- Min number of arcs around one node
- Random upper bound
- Distribution of nodes
- Interactive network modification
- Random costs

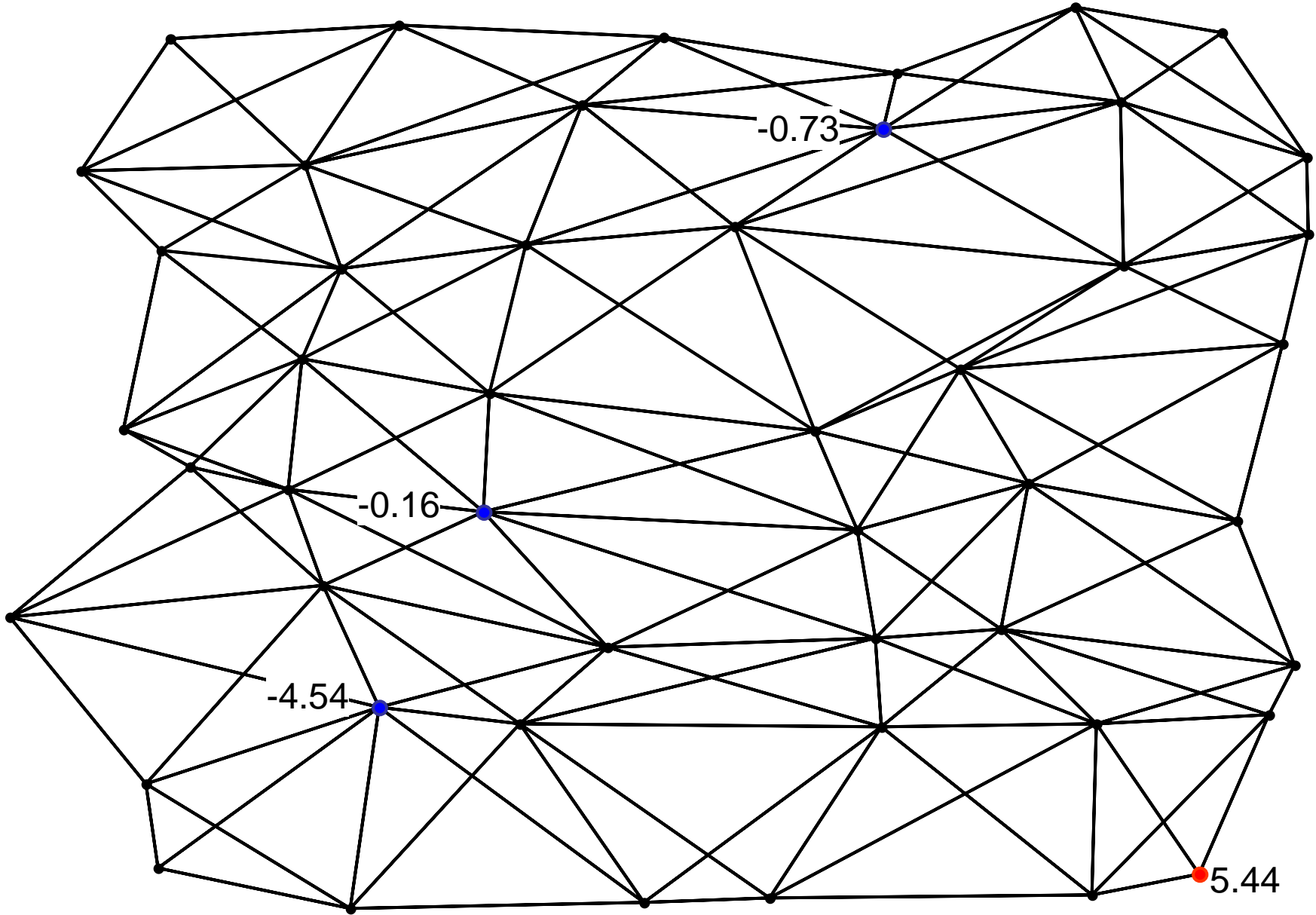
The algorithm provides:

- Number of nodes
- Upper bound of capacity
- A_{eq} matrix
- Balanced production/consumption at the sources and sinks

[Aeq,beq,lb,ub,c]=simsys_sparse(100);

Solution in Matlab: $x = \text{linprog}(c,[],[],Aeq,beq,lb,ub)$

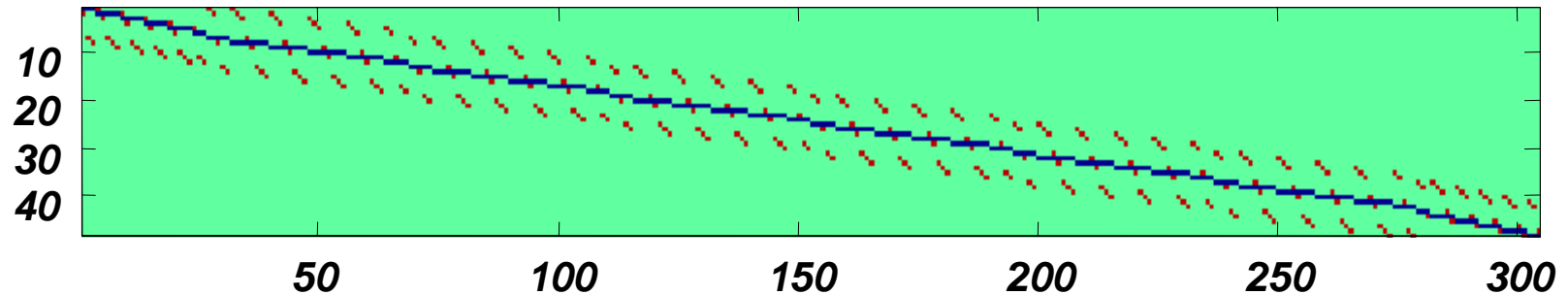
RANDOMLY GENERATED NETWORK



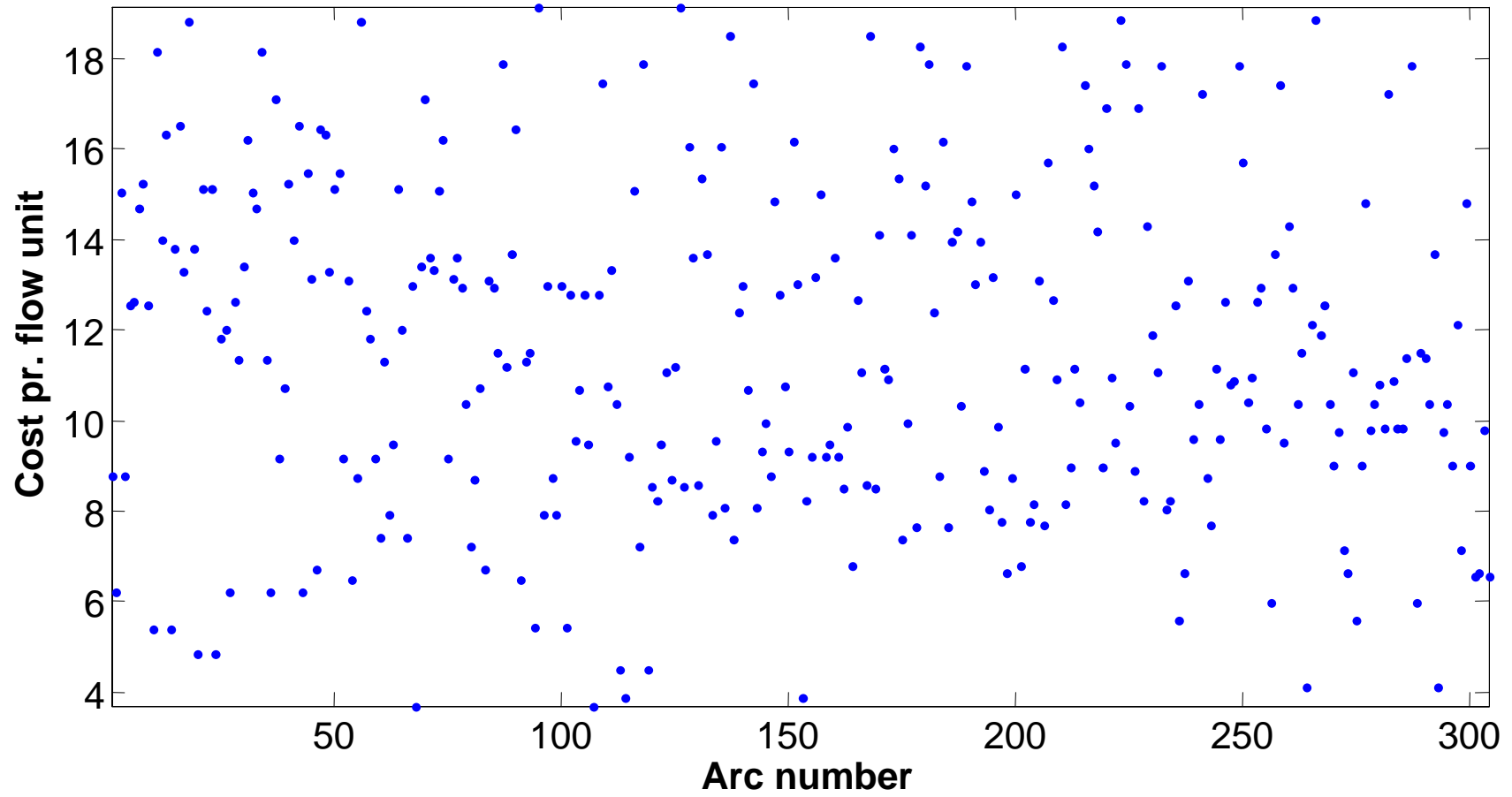
The LP-problem:

- Number of arcs: 304
- Lower bounds: 0
- Upper bounds: -
- Equality constraints: 48

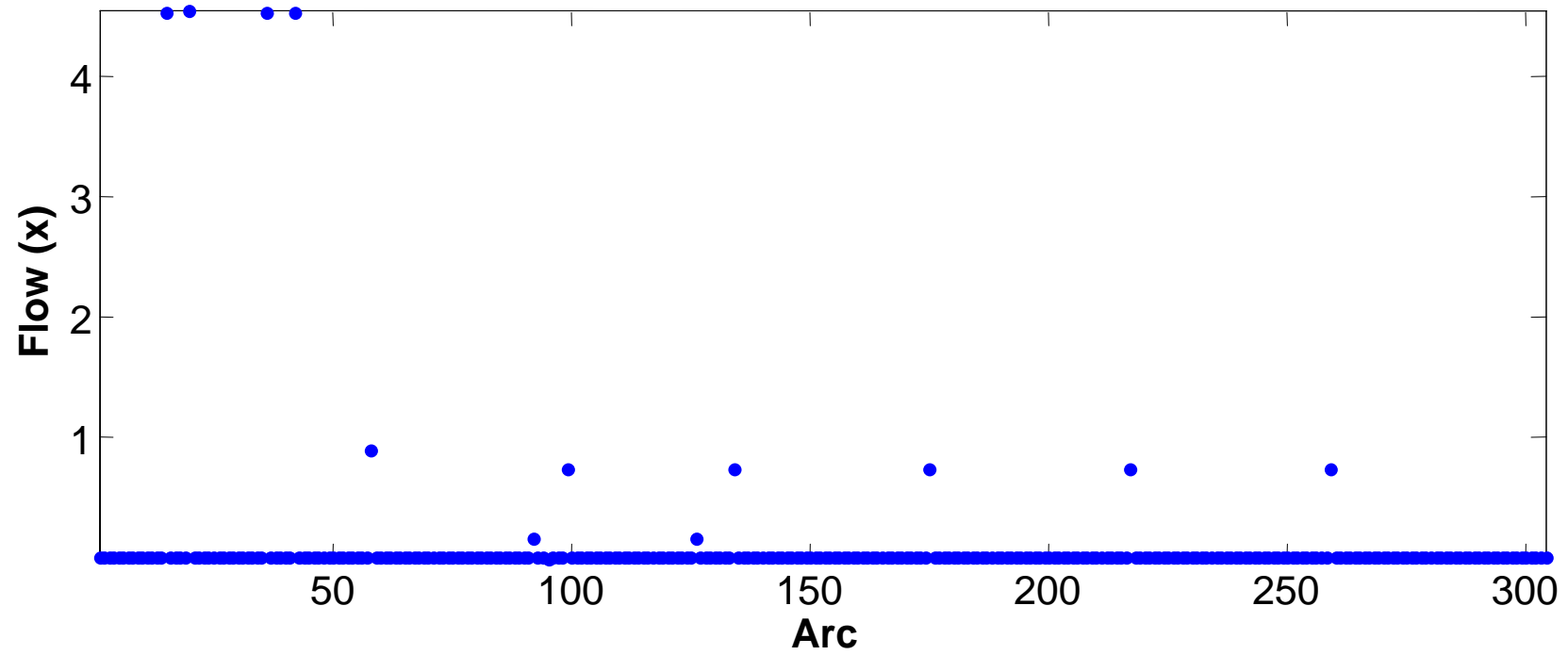
A_{eq} -matrix:

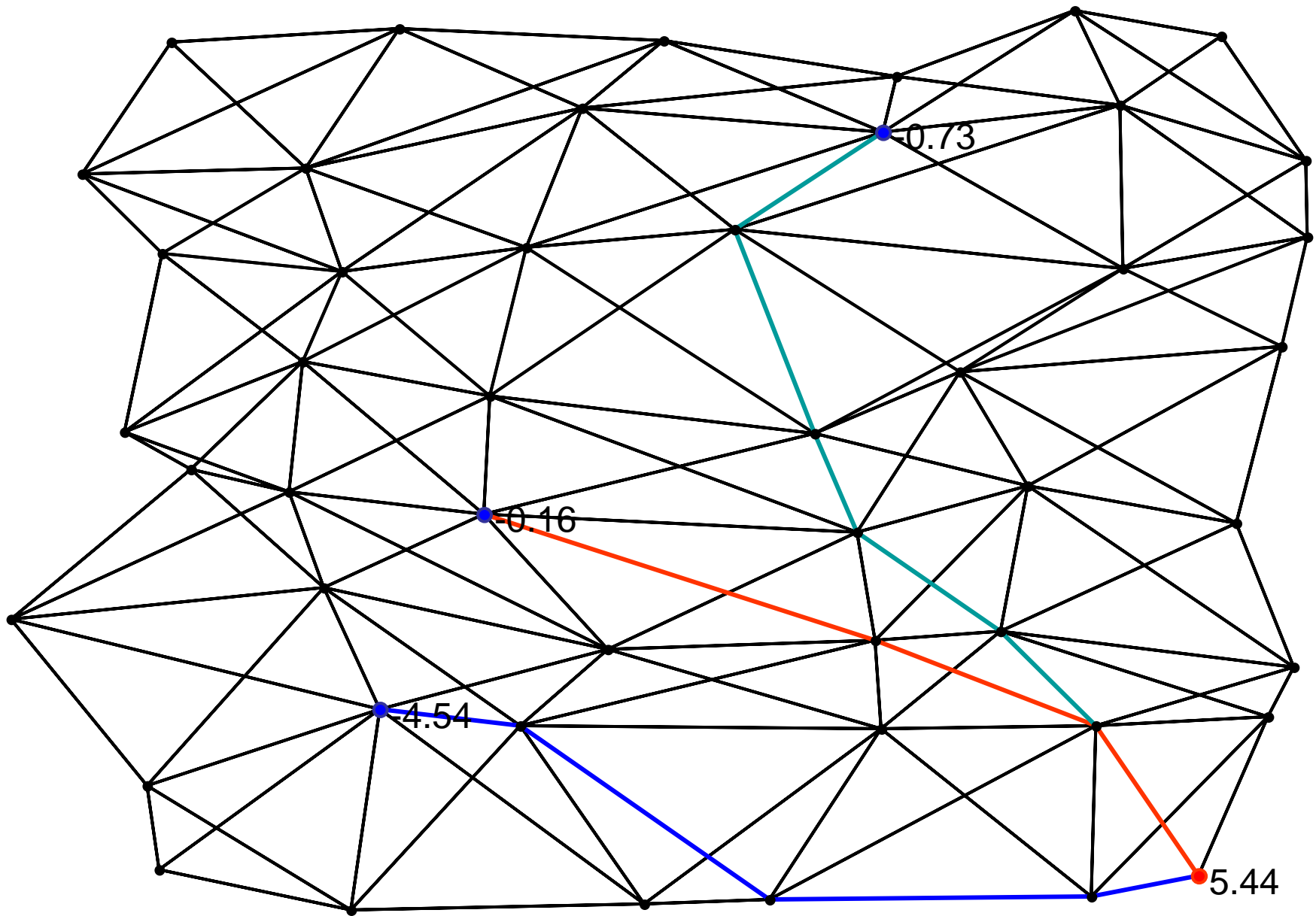


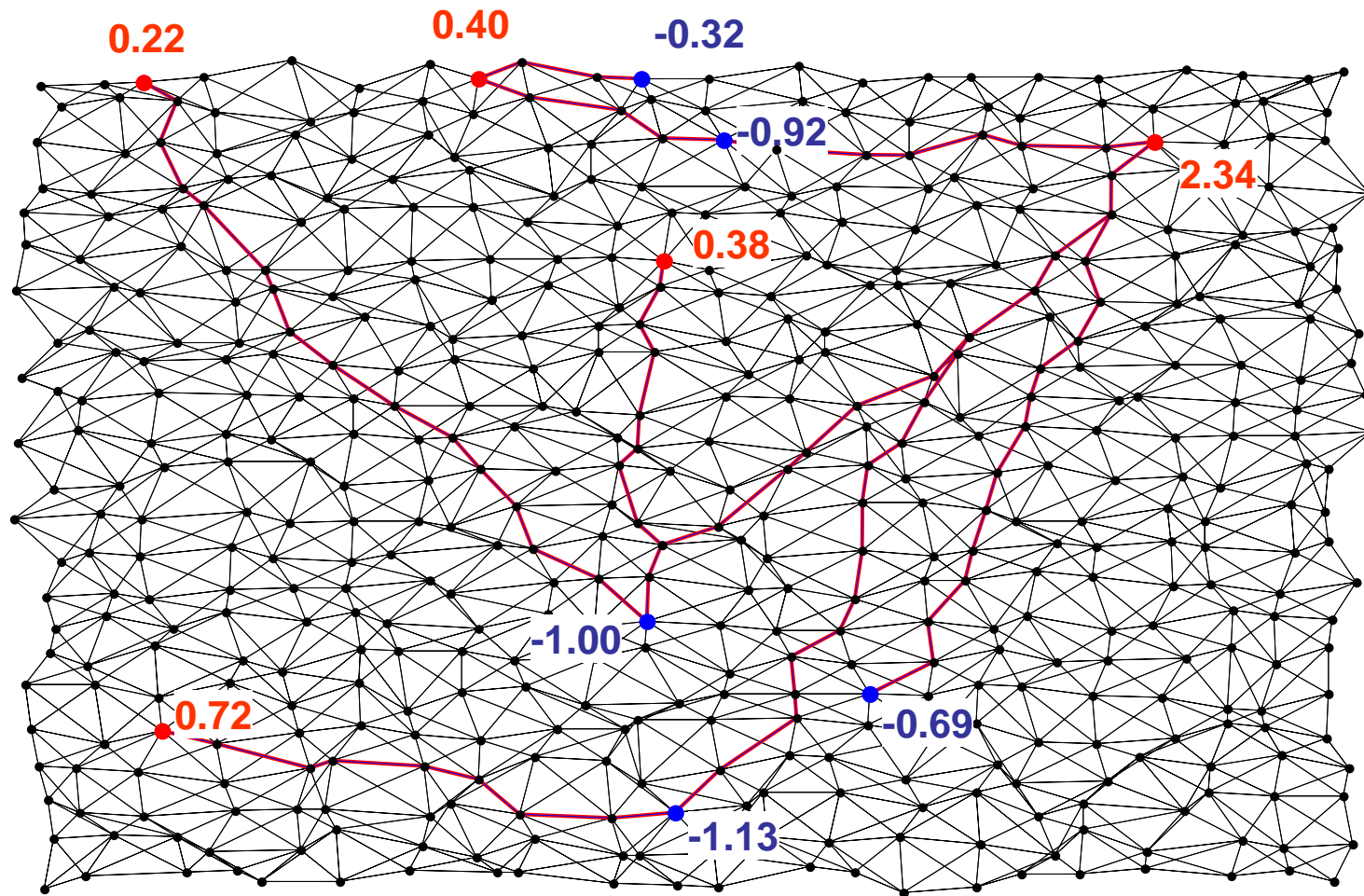
Costs



LP solution

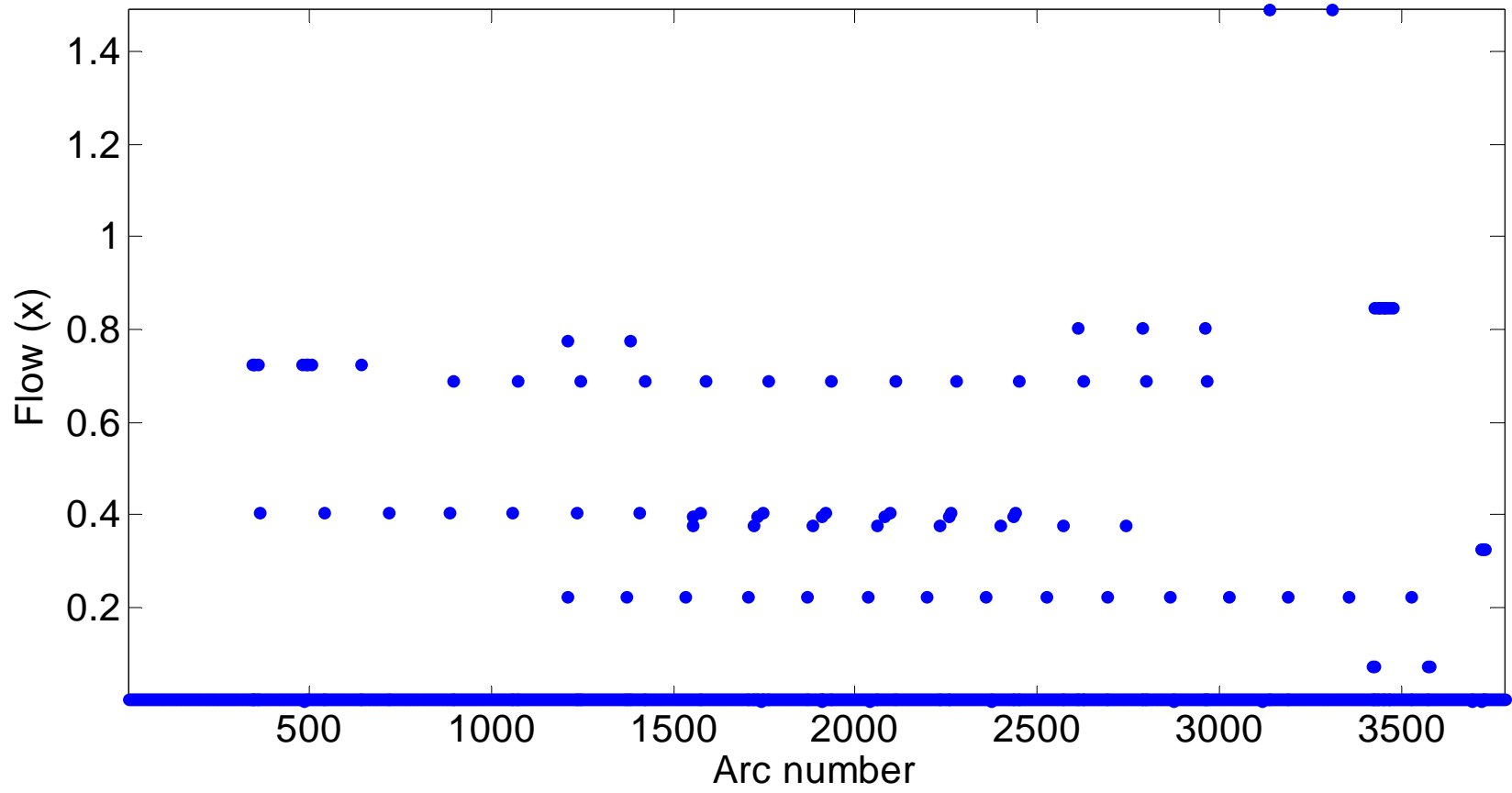






$$\dim(x) = 3782$$

$$\dim(A_{eq}) = 506 \times 3782$$



Practical Optimization: A Gentle Introduction

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<http://www.sce.carleton.ca/faculty/chinneck/po.html>

(*very soft introduction* 😊)