QUADRATIC PROGRAMMING IN MATLAB

quadprog

$$\min_x \left\{ \frac{1}{2} x' H x + f' x \right\}$$

$$Ax \le b$$

$$A_{eq} x = b_{eq}$$

$$lb \le x \le ub$$

Syntax:

x = quadprog(H,f,A,b)
x = quadprog(H,f,A,b,Aeq,beq)
x = quadprog(H,f,A,b,Aeq,beq,lb,ub)
x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0)
x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)
[x,fval] = quadprog(...)
[x,fval,exitflag] = quadprog(...)
[x,fval,exitflag,output] = quadprog(...)
[x,fval,exitflag,output,lambda] = quadprog(...)

Small-scale problem

$$\min_x \left\{ \frac{1}{2} x' H x + d' x \right\},$$

$$Ax \le b, 0 \le x$$

$$H = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, d = \begin{bmatrix} -2 \\ -6 \end{bmatrix},$$

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

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H = [1 -1; -1 2]
d = [-2; -6]
A = [1 1; -1 2; 2 1]
b = [2; 2; 3]
lb = zeros(2,1)

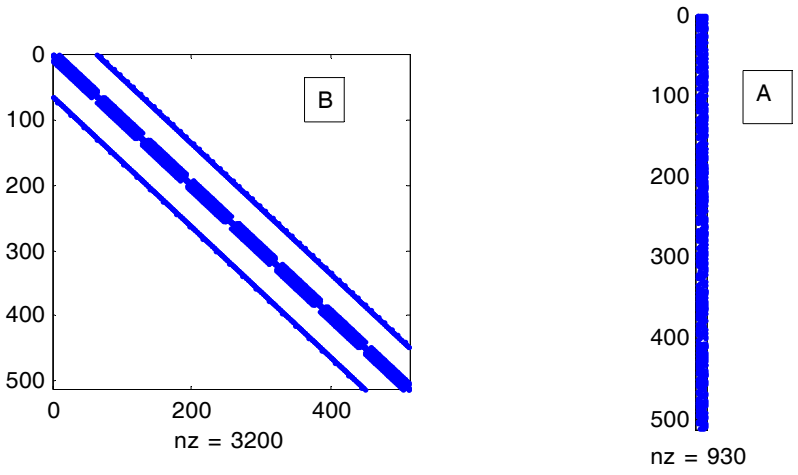
[x,fval,exitflag,output,lambda] = ...
quadprog(H,d,A,b,[],[],lb)

x =  ( 0.6667, 1.3333)
fval =  -8.2222
output =
      iterations:    3
      algorithm:    'medium-scale: active-set'
```

LARGE SCALE PROBLEM

Dense, but structured Hessian:

$$H = B + AA'$$



We must avoid forming *H*!

Special software for forming $H*W$:

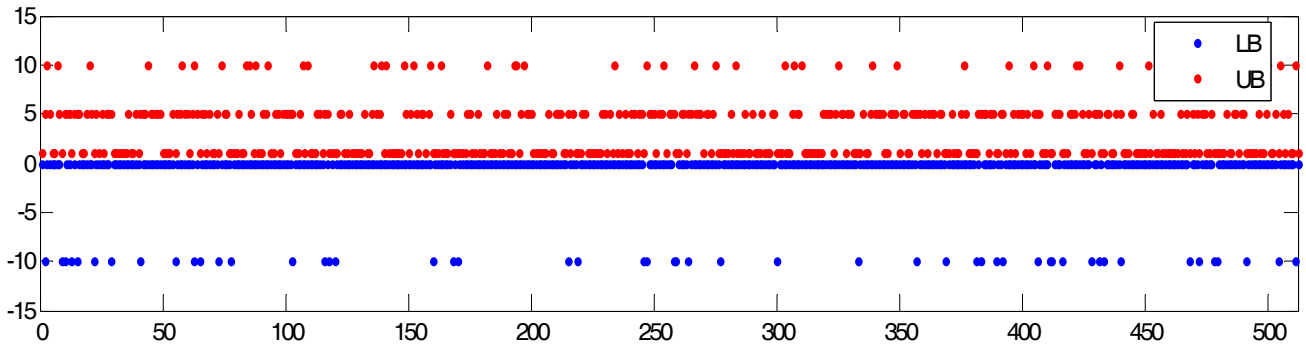
W = qpbox4mult(H,Y)

(This is simple: W = B*Y + A*(A'*Y);)

Problem entered by

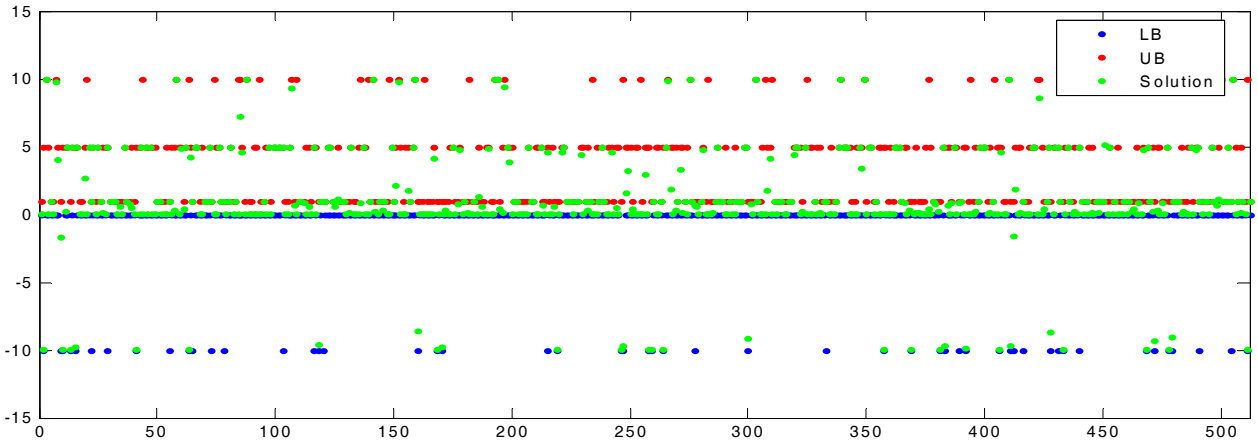
load qpbox4.mat

A	512x10	11204	double array (sparse)
B	512x512	40452	double array (sparse)
d	512x1	4096	double array
lb	512x1	4096	double array
ub	512x1	4096	double array
xstart	512x1	4096	double array



Optimization terminated: relative function value changing by less than sqrt(OPTIONS.TolFun), no negative curvature detected in current trust region model and the rate of progress (change in f(x)) is slow.

fval = -1.0538e+003
exitflag = 3
output =
    iterations: 18
    algorithm: 'large-scale: reflective trust-region'
    firstorderopt: 0.0043
    cgiterations: 30
x = ...


LB
UB
Solution
15
10
5
0
-5
-10
-15
0
50
100
150
200
250
300
350
400
450
500