# Equivalence relations

Let $X$ be a set. An equivalence relation on $X$ is a subset $E$ of of the set $X \times X = \{(x_1, x_2) | x_1, x_2 \in X\}$ satisfying the following three conditions

(reflexivity) $(x, x) \in E$ for all $x \in X$

(symmetry) If $(x_1, x_2) \in E$ then $(x_2, x_1) \in E$

(transitivity) If $(x_1, x_2) \in E$ and $(x_2, x_3) \in E$ then $(x_1, x_3) \in E$

We often write $x_1 \sim x_2$ instead of $(x_1, x_2) \in E$.
Given an equivalence relation $E$ on a set $X$ we may for each $x \in X$ define the equivalence class of $x$ to be the subset $[x] = \{y \in X | x \sim y\}$.
This divides $X$ into a collection of nonempty, mutually disjoint subsets.
The set of equivalence classes is written $X/\sim$, and we have a surjective function $X \to X/\sim$ sending $x \in X$ to its equivalence class $[x]$.