1.14

Ex of sequences:

LLLLLW
LLLLLLW
W
LW

Studying the problem throw by throw is hard, but after 2 throws one of the following happen

1. Player 1 wins
2. Player 2 wins
3. We are back in the same state

This means we can divide the sequences of throws into groups of 2 and 2 throws

Ex.

$\boxed{LL}\boxed{LL}\boxed{LW}$
$\boxed{W}$

Each group is

$$\left.\begin{array}{ll} \boxed{W} & \text{P1 wins, } P = p \\ \boxed{LW} & \text{P2 wins, } P = (1-p)p \\ \boxed{LL} & \text{No one wins, } \underline{\text{repeat}} \end{array}\right\} \text{Disjoint}$$

This is exactly the situation in Problem 1.12.

$$\text{Prob}(\text{P1 wins}) = \frac{p}{p+p(1-p)} = \frac{1}{2-p}$$

$$\text{Prob}(\text{P2 wins}) = 1 - \frac{1}{2-p} = \frac{1-p}{2-p}$$

See "P1_14.R" for code to try.

P. 1.39

A = "Employee is From store A"
B = " ———— " ——— B"
C = " ———— " ——— C"
W = "Employee is Female"

Choosing employee randomly gives

$P(A) = \frac{50}{225}$, $P(B) = \frac{75}{225}$, $P(C) = \frac{100}{225}$

Given in problem

$P(W|A) = 0.5$, $P(W|B) = 0.6$, $P(W|C) = 0.7$

We want $P(C|W)$

Use Bayes' law

$$P(C|W) = \frac{P(W|C) \cdot P(C)}{P(W)}$$

Law of total prob gives

$$P(W) = P(A)P(W|A) + P(B)P(W|B) + P(C)P(W|C)$$
$$\approx 0.622$$

Thus

$$P(C|W) = \frac{0.7 \cdot \frac{100}{225}}{0.622} = 0.5$$

P 2.42

m different coupons

Each coupon is randomly from $\{1, \dots, m\}$

Let X = "Number of coupons needed to collect all"

Want $E[X]$

Probably seen this in the Stastics course.

Idea:

- Decompose the difficult R.V. X into simpler R.V.s
- Calculate the expected value of X through the expected values of the simpler parts.

The hint says to decompose X into a sum of geometric R.V.s.

Step 1: THINK!

- What type of behaviour in this problem is "geometrical"?
- Time until a certain coupon is received OR
  Time until a NEW coupon is received!

Step 2: Constract formal description

X_i = "Time from coupon $i-1$ is received to ith unique is received"

$$X_i \sim geo\left(\frac{n-i+1}{n}\right)$$

And $X = X_1 + X_2 + \dots + X_n$

Step 3: Calc. $E[X]$

$$E[X] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} \frac{n}{n-i+1} = n\left[\frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{1}\right]$$

$$= n\left[1 + \frac{1}{2} + \dots + 1/n\right]$$

$$\sim n\log(n)$$

P2.34

Was supposed to be on exercise sheet.

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Must have

$$\int_{\mathbb{R}} f(x)\,dx = 1$$

$$\int_0^2 c(4x - 2x^2)\,dx = 1$$

$$c\left[2x^2 - \frac{2}{3}x^3\right]_0^2 = 1$$

$$c\left(\frac{8}{3}\right) = 1 \Rightarrow \underline{\underline{c = 3/8}}$$

b) $P(\frac{1}{2} < X < 3/2) = \int_{1/2}^{3/2} f(x)\,dx = \frac{3}{8}\left[2x^2 - \frac{2}{3}x^3\right]_{1/2}^{3/2} = \underline{\underline{0.6875}}$

P 1.17

$\{1, \dots, r\}$ outcomes

with probabilities $p_1, p_2, \dots, p_r$, s.t. $\sum_{i=1}^{r} p_i = 1$

n experiments

Independent

Let X_i = "# i occurs"

What is $P(X_1 = x_1, \dots, X_r = x_r)$?

Including order: $p_1^{x_1} \cdots p_r^{x_r}$

Need to know how many orderings exist

Couple of ways of doing this

1. In total $n!$ ways of reordering n elements

   Ex $1234 \to 1342 \to \cdots$

   But some elements may be equal so there is <u>not</u> $n!$ unique ones.

   · For each i there are $x_i!$ ways to reoder the i-s such that the ordering stays the same.

   $\Rightarrow$ Total num is $\dfrac{n!}{x_1! \cdots x_n!}$

   Or $P = \dfrac{n!}{x_1! \cdots x_n!} \cdot p_1^{x_1} p_2^{x_2} \cdots p_n^{x_n}$

2. This exactely what the mutinomial coefficient describes $\binom{n}{x_1, \dots, x_n}$.

R stuff

```
> x=rbeta(n, shape1, shape2)
> hist(x, breaks =50)
> x=seq(0,1, length.out =50)
> lines(x, dbeta(x, shape1, shape2)
```