

1.14

Ex of sequences:

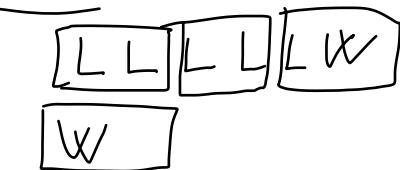
L L L L L W
 L L L L L L W
 W
 LW

Studying the problem throw by throw is hard, but after 2 throws one of the following happen

1. Player 1 wins
2. Player 2 wins
3. We are back in the same state

This means we can divide the sequences of throws into groups of 2 and 2 throws

Ex.



Each group is

[W]	P1 wins, $P = p$	Disjoint
[L W]	P2 wins, $P = (1-p)p$	
[L L]	No one wins, <u>repeat</u>	

This is exactly the situation in Problem 1.12.

$$\text{Prob}(P1 \text{ wins}) = \frac{p}{p + p(1-p)} = \frac{1}{2-p}$$

$$\text{Prob}(P2 \text{ wins}) = 1 - \frac{1}{2-p} = \frac{1-p}{2-p}$$

See "P1_14.R" for code to try.

P. 1.39

$$A = \text{"Employee is From store A"} \\ B = \text{"Employee is From store B"} \\ C = \text{"Employee is Female"} \\ W = \text{"Employee is Female"}$$

Choosing employee randomly gives

$$P(A) = \frac{50}{225}, P(B) = \frac{75}{225}, P(C) = \frac{100}{225}$$

Given in problem

$$P(W|A) = 0.5, P(W|B) = 0.6, P(W|C) = 0.7$$

We want $P(C|W)$

Use Bayes' law

$$P(C|W) = \frac{P(W|C) \cdot P(C)}{P(W)}$$

Law of total prob gives

$$P(W) = P(A)P(W|A) + P(B)P(W|B) + P(C)P(W|C) \\ \approx 0.622$$

Thus

$$P(C|W) = \frac{0.7 \cdot \frac{100}{225}}{0.622} = 0.5$$

P 2.42

m different coupons

Each coupon is randomly from $\{1, \dots, m\}$

Let $X = \text{"Number of Coupons needed to collect all"}$

Want $E[X]$

Probably seen this in the Statistics course.

Idea:

- Decompose the difficult R.V. X into simpler R.V.s
- Calculate the expected value of X through the expected values of the simpler parts.

The hint says to decompose X into a sum of geometric R.V.s.

Step 1: THINK!

- What type of behaviour in this problem is "geometrical"?
- Time until a certain coupon is received OR
Time until a NEW coupon is received!

Step 2: Construct Formal description

$X_i = \text{"Time from coupon } i-1 \text{ is received to } i^{\text{th}}$
 $\text{unique is received"}$

$$X_i \sim \text{geo}\left(\frac{n-i+1}{n}\right)$$

And $X = X_1 + X_2 + \dots + X_n$

Step 3: Calc. $E[X]$

$$\begin{aligned} E[X] &= \sum_{i=1}^n E[X_i] = \sum_{i=1}^n \frac{n}{n-i+1} = n \left[\frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{1} \right] \\ &= n \left[1 + \frac{1}{2} + \dots + \frac{1}{n} \right] \\ &\sim n \log(n) \end{aligned}$$

P 2.34

Was supposed to be on exercise sheet.

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Must have

$$\int_{\mathbb{R}} f(x) dx = 1$$

$$\int_0^2 c(4x - 2x^2) dx = 1$$

$$c \left[2x^2 - \frac{2}{3}x^3 \right]_0^2 = 1$$

$$c \left(\frac{8}{3} \right) = 1 \Rightarrow c = \underline{\underline{\frac{3}{8}}}$$

$$b) P\left(\frac{1}{2} < x < \frac{3}{2}\right) = \int_{1/2}^{3/2} f(x) dx = \underline{\underline{\frac{3}{8} \left[2x^2 - \frac{2}{3}x^3 \right]_{1/2}^{3/2}}} = \underline{\underline{0.6875}}$$

P1.17

$\{1, \dots, r\}$ outcomes

with probabilities p_1, p_2, \dots, p_r , s.t. $\sum_{i=1}^r p_i = 1$
 n experiments

Independent

Let $X'_i = \text{"# } i \text{ occurs"}$

What is $P(X_1 = x_1, \dots, X_r = x_r)$?

Including order: $p_1^{x_1} \cdots p_r^{x_r}$

Need to know how many orderings exist

Couple of ways of doing this

1. In total $n!$ ways of reordering n elements

Ex $1234 \rightarrow 1342 \rightarrow \dots$

But some elements may be equal so there
is not $n!$ unique ones.

For each i there are $x_i!$ ways to
reorder the i -s such that the ordering stays the
same.

\Rightarrow Total num is $\frac{n!}{x_1! \cdots x_n!}$

$$\text{Or } P = \frac{n!}{x_1! \cdots x_n!} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdots p_n^{x_n}$$

2. This exactly what the multinomial
coefficient describes $\binom{n}{x_1, \dots, x_n}$.

R stuff

```
> x=rbeta(n, shape1, shape2)
> hist(x, breaks =50)
> x=seq(0,1, length.out =50)
> lines(x, dbeta(x, shape1, shape2))
```