1 Exercises from the book

4.16, 4.18 and 4.32.

2 Exercise 1

Given a homogeneous Markov chain, where the transition matrix $\mathbf{P}$ depends on a parameter p given by

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0.2 & p & 0 & 0.8-p \\ 0.3 & 0.7 & 0 & 0 \\ 0 & 0.1 & 0.1 & 0.8 \\ 0.1 & p & 0.1 & 0.8-p \end{pmatrix} \end{array}$$

For which value of p is the Markov chain not irreducible?

3 Exercise 2

Consider a Markov chain with state space $\Omega = 1, 2, 3, 4, 5, 6$ and transition matrix

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix} \end{array}$$

Determine:

a) The period of each state

b) Which states are transient

c) Which states are ergodic

d) The equivalence classes.

4 Exercise 3

Consider a Markov chain whose transition probability matrix is given by

$$\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

a) Starting in state 1, determine the probability that the Markov chain is absorbed in state 0.

b) Determine the mean time of absorption given the process starts in state 1.