

TMA4265 Stochastic Processes

Week 36 – Exercises

Problem 1: Exercises from the book

3.8 and 3.24.

Problem 2: Changes in stock prices

Let Z be a random variable that denotes the price change for one stock in a specific company from the stock exchange closes one day till it closes the following day. Each day there may be multiple trades. One possible model is given by

$$Z = X_0 + X_1 + \dots + X_N,$$

where X_0, X_1, \dots, X_N are independent random variables with normal distributions with mean 0 and variance σ^2 , that are independent of N which has a Poisson distribution with rate ν . X_0 is the initial price change when the stock exchange opens, X_i , for $i > 0$, is the price change after trade i and N is the total number of trades while the stock exchange is open.

1. Calculate the variance of Z .

Problem 3: Joint distribution

The joint probability mass function of two discrete random variables X and Y is given by

$$p(x, y) = \exp(-2\lambda) \frac{\lambda^{x+y}}{x! y!} \quad \text{for } x, y \in \{0, 1, \dots\}$$

with $\lambda > 0$.

1. Compute the marginal probability mass functions $p_X(x)$ and $p_Y(y)$. Compute the covariance of X and Y .
2. Compute the conditional probability mass function of $X|X+Y$.
3. Compute the covariance of $X+Y$ and $X-Y$.

Problem 4: Expectation

1. We roll a standard die over and over. What is the expected number of rolls until the first pair of consecutive sixes appears?