## FLT and FLW - a replication

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Harald Henche-Olsen (HHO) has, on a frivolous personal webpage (http://www.math.ntnu.no/~hanche/blog/trell/trell-lft.pdf; + ~hanche/blog/trell/) poured out some malicious misrepresentation of an archetypal proof and reproof of FLT (Fermat's Last Theorem, which essentially states that $X^{n}+Y^{n} \neq Z^{n}$ ). ${ }^{1}$ The both subjective and apocryphal nature of his envious confabulations should make them self-abortive as such. However, they are lodged on an official University homesite so it is obligatory that the same is open for a formal rejoinder, which will expose that for matter-of-fact and intellectual reasons they run for becoming his scientific FLW (Famous Last Words) as well. To that end, the proper replication to the said forger is a straightforward replication of the actual FLT demonstration he belies.

This is performed by the (re?)invention ${ }^{2}$ of an easily testable and fully reproducible structural embodiment, or 'dummy' (Fig. 1), which in a classic mathematical Deus ex Machina way (compare e.g. Eratosthenes' sieve ${ }^{3,4}$ ) stepwise executes and extracts an exact facsimile of the basic $X^{3}+Y^{3}=Z^{3}$ (anti)thesis of FLT for that power. In serial reduplications of its cogwheel mechanism it likewise serves as the principle FLT mould and generator in all


Fig.1. Stepwise coating of $Z^{3}$, illustrated by a modification of a painting by $\overline{\text { Victor Vasarely, which shows (a) by an extract and dissection of } 3^{3} \text { how this (in }}$ that projection) is grown by a $3^{2}$ roof plus a $3 \times 2$ front plus a $2^{2}$ side according to ascending $Z^{2}+Z(Z-1)+(Z-1)^{2}$ difference formula, and $(b)$ by the optical illusion in the $6^{3}$ block how the function works both in the + and - directions.
degrees of n . Especially for the young, code-cracking minds, free from doctrinaire or tribal bonds, it should therefore, in the words of Roger Penrose ${ }^{5}$ be a useful "toy model" for rational real space experiment and exploration (and a concrete challenge also for the prohibitionists who must now bolster up their hitherto lazy anathema by objective arguments).

This virtual FLT dynamo is idio-automorphic because by equally classic ancient provenience ${ }^{4,6}$ its elementary building brick, or tessella, is a ground unit cube. By its eccentric placing and growth from an arbitary origin corner of the Cartesian co-ordinate system the device quite directly, exactly and exhaustively produces what it purports to produce, namely, to the ground order, all successive whole-number cubes, or three-folds that there are.

And it does it in the mode that every new 3 -fold, hence $Z^{3}$, in a successive brick-laying mode is formed, or tessellated ${ }^{4}$, from the underlying, hence $\mathrm{X}^{3}$, by the primarily single-layer growth, or difference $(\Delta)$ stratum which is thus the hypothetical $\mathrm{Y}^{3}$ term in the bottom-line FLT equation. Very simple and still adequate and apt; not even HHO could deny that. Instead he had to cheat at the ensuing powers, shallowly failing to see - or acknowledge - the significance of what he yet admits, that the derived mono-layer growth differential when realistically expressed in terms of Z is generalisable to all other degrees beyond $\mathrm{n}=3$. At the $\mathrm{n}=3$ level it obviously is:

$$
\mathrm{Z}^{2}+\mathrm{Z}(\mathrm{Z}-1)+(\mathrm{Z}-1)^{2}
$$

but when $\mathrm{n}=4$, by identical expansion raises to

$$
Z^{3}+Z^{2}(Z-1)+Z(Z-1)^{2}+(Z-1)^{3}
$$

when $n=5$

$$
\mathrm{Z}^{4}+\mathrm{Z}^{3}(\mathrm{Z}-1)+\mathrm{Z}^{2}(\mathrm{Z}-1)^{2}+\mathrm{Z}(\mathrm{Z}-1)^{3}+(\mathrm{Z}-1)^{4}
$$

and so on, ad infinitum, and as possible to express, exactly and exhaustively by the geometrical series:

$$
\mathrm{Z}^{\mathrm{n}-1}+\mathrm{Z}^{\mathrm{n}-2}(\mathrm{Z}-1)+\mathrm{Z}^{\mathrm{n}-3}(\mathrm{Z}-1)^{2}+\ldots . \mathrm{Z}^{2}(\mathrm{Z}-1)^{\mathrm{n}-3}+\mathrm{Z}(\mathrm{Z}-1)^{\mathrm{n}-2}+(\mathrm{Z}-1)^{\mathrm{n}-1}
$$

HHO triumphs that this is indeed a very basic geometric series. But the real triumph, that HHO doesn't mention or even see the significance of, is that in the described way it is undeniably and unalienably an intrinsic function and yield of the applied reproduction and conveys its operations and outcome in
all dimensions so that indeed the $\mathrm{X}^{3}+\mathrm{Y}^{3}=\mathrm{Z}^{3}$ Deus ex Machina is the escalating $X^{n}+Y^{n}=Z^{n}$ Deus ex Machina as well.

And it doesn't take higher mathematical sophistication to realise that the only whole-number solution in any degree $\geq 3$ of its fundamental one-layer growth formula equals 1 . When $\mathrm{n}=2$ it is different, though. Why? Also and at the first order because the initiating mono-layer growth stratum is composed by first-degree terms: $Z+(Z-1)$. And similarly, two-layer growth can be expressed in first degree form: $2[\mathrm{Z}+(\mathrm{Z}-2)]$, as can 3 layers (see the front side of Fig. 1b): $3[Z+(Z-3)], 4$ layers: $4[Z+(Z-4)] \ldots$, and so on infinitely.

Any alert and unbiased student at sole inspection of the FLT automate immediately conceives that at the $n=3$ level it generates all $Z^{3}$ sums as the combination of the innermost $\mathrm{X}^{3}$ term in the origin corner and the $\mathrm{Y}^{3}$ cap (which is quite accurately referred to as $\Delta$ ). And it is equally obvious that this stratum may consist of more than one layer, and so the whole remaining enterprise is about finding a general formula to this adjustment of the geometrical series that even HHO had to concede was as such adequately identified with the Y term of the binary addition.

So, when $\mathrm{n}=3$ and there is one growth layer alone:

$$
\left[Z^{2}+Z(Z-1)+(Z-1)^{2}\right]=3 Z^{2}-3 Z+1
$$

when there are two layers:

$$
\left[Z^{2}+Z(Z-1)+(Z-1)^{2}\right]+\left[(Z-1)^{2}+(Z-1)(Z-2)+(Z-2)^{2}\right]=6 Z^{2}-12 Z+8
$$

three layers:

$$
\left[\mathrm{Z}^{2}+\mathrm{Z}(\mathrm{Z}-1)+(\mathrm{Z}-1)^{2}\right]+\left[(\mathrm{Z}-1)^{2}+(\mathrm{Z}-1)(\mathrm{Z}-2)+(\mathrm{Z}-2)^{2}\right]+\left[(\mathrm{Z}-2)^{2}+(\mathrm{Z}-2)(\mathrm{Z}-3)+(\mathrm{Z}-3)^{2}\right]=
$$

$$
9 Z^{2}-27 Z+27
$$

and four layers:

$$
12 Z^{2}-48 Z+64, \text { and so on }
$$

The next step is to generalise this expression to a function of the number of layers, $\lambda$ (nota bene, HHO, number of layers, hence "any $\lambda$ ": the crux of your counterfeit). From the above this function is obtained as a simple geometrical series, too. When $\mathrm{n}=3$, the general $\mathrm{Y}^{3}$ formula in question is:

$$
Y^{3}=3 \lambda Z^{2}-3 \lambda^{2} Z+\lambda^{3}
$$

and it is clear that the only whole-number solution is when (any) $\lambda=$ (any) Z, for then, posed as an equation, $\mathrm{Y}^{3}=3 \lambda^{3}-3 \lambda^{3}+\lambda^{3}=\lambda^{3}=Z^{3}$, with only wholenumber third root $=\mathrm{Z}$. Hence $\mathrm{Y}=\mathrm{Z}$ and FLT proved for $\mathrm{n}=3$. Circle arguing? No, straight and undeniable, yet very simple as truth and reality as such.

And it is equally undeniable that when $n=4$ the general $\mathrm{Y}^{4}$ formula by the same deduction turns out as:

$$
Y^{4}=4 \lambda Z^{3}-6 \lambda^{2} Z^{2}+4 \lambda^{3} Z-\lambda^{4},
$$

with sole integer solution when $\lambda=Z: Y^{4}=2 \lambda^{4}-\lambda^{4}=\lambda^{4}\left(=Z^{4}\right)$, fourth root $=Z$.

And $\mathrm{n}=5$ :

$$
\left(Y^{5}\right)=5 \lambda Z^{4}-10 \lambda^{2} Z^{3}+10 \lambda^{3} Z^{2}-5 \lambda^{4} Z+\lambda^{5} \text {, finally yielding } Y^{5}=0+\lambda^{5}=\lambda^{5}
$$

$n=6$ :

$$
6 \lambda Z^{5}-15 \lambda^{2} Z^{4}+20 \lambda^{3} Z^{3}-15 \lambda^{4} Z^{2}+6 \lambda^{5} Z-\lambda^{6} ; \ldots . Y^{6}=(6-15+20-15+6-1) \lambda^{6}=\lambda^{6}
$$

$\mathrm{n}=7$ :
$7 \lambda Z^{6}-21 \lambda^{2} Z^{5}+35 \lambda^{3} Z^{4}-35 \lambda^{4} Z^{3}+21 \lambda^{5} Z^{2}-7 \lambda^{6} Z+\lambda^{7} ; \quad Y^{7}=(6-21+35-35+21-7+1) \lambda^{7}=\lambda^{7}$
$\mathrm{n}=8$ :
$8 \lambda Z^{7}-28 \lambda^{2} Z^{6}+56 \lambda^{3} Z^{5}-70 \lambda^{4} Z^{4}+56 \lambda^{5} Z^{3}-28 \lambda^{6} Z^{2}+8 \lambda^{7} Z-\lambda^{8} ; \quad Y^{8}=\ldots \ldots \ldots . . .2 \lambda^{8}-\lambda^{8}=\lambda^{8}$
$\mathrm{n}=9$ :
$9 \lambda Z^{8}-36 \lambda^{2} Z^{7}+84 \lambda^{3} Z^{6}-126 \lambda^{4} Z^{5}+126 \lambda^{5} Z^{4}-84 \lambda^{6} Z^{3}+36 \lambda^{7} Z^{2}-9 \lambda^{8} Z+\lambda^{9} ; \quad Y^{9} \ldots \ldots=\lambda^{9}$
$\mathrm{n}=10$ :
$10 \lambda Z^{9}-45 \lambda^{2} Z^{8}+120 \lambda^{3} Z^{7}-210 \lambda^{4} Z^{6}+252 \lambda^{5} Z^{5}-210 \lambda^{6} Z^{4}+120 \lambda^{7} Z^{3}-45 \lambda^{8} Z^{2}+10 \lambda^{9} Z-\lambda^{10} ;$

$$
Y^{10}=\ldots . .2 \lambda^{10}-\lambda^{10}=\lambda^{10}
$$

And so it goes on. Already now it is entirely evident by the iterated symmetry of the serial expansion, that this general formula acts in every higher n and exclusively when (any) $\mathrm{Z}=$ (any) $\lambda$ always yields the only possible whole-number solution $\mathrm{Y}^{\mathrm{n}}=\lambda^{\mathrm{n}}\left(=\mathrm{Z}^{\mathrm{n}}\right)$. When $\mathrm{n}=2$ it is as mentioned different because while there is still a valid universal $\mathrm{Y}^{2}=2 \lambda \mathrm{Z}-\lambda^{2}$ equation this is not unique since there are also these $\mathrm{Z}+(\mathrm{Z}-1), 2[\mathrm{Z}+(\mathrm{Z}-2)], 3[\mathrm{Z}+(\mathrm{Z}-3)]$ etc. firstdegree alternative equations which are not locked by terms of higher dignity.

But from $\mathrm{n}=3$ there is an absolute, i.e. both universal and unique algorithm direct from the 'magical triangle' of the coefficients of the successive $\lambda$ series expansion. Re-stating them in the $\mathrm{n}=3$ to $\mathrm{n}=10$ levels here accounted for in detail, they are:

| $\mathrm{n}=3$ : | $3-3$ |  |  |  |  |  |  |  | (+1) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}=4$ | 4 | -6 | +4 |  |  |  |  |  | (-1) |
| $\mathrm{n}=5$ | 5 | -10 | +10 | -5 |  |  |  |  | (+1) |
| $\mathrm{n}=6$ | 6 | -15 | +20 | -15 | +6 |  |  |  | (-1) |
| $\mathrm{n}=7$ | 7 | -21 | +35 | -35 | +21 | -7 |  |  | (+1) |
| $\mathrm{n}=8$ | 8 | -28 | +56 | -70 | +56 |  | +8 |  | (-1) |
| $\mathrm{n}=9$ | 9 | -36 | +84 | -126 | +126 | -84 | +36 | -9 | (+1) |
| $\mathrm{n}=10$ | 10 | -45 | +120 | -210 | +252 | -210 | +120 | $-45+10$ | (-1) |

.
ad infinitum

There are a lot of lawbound symmetries in that diagonal matrix, enabling the specification of the general $\Delta$ formula in any level of $n$ (for instance, $\mathrm{n}=987654321$ if so be). One immediately sees that the first column increases by steps of one, the second by the series $3,4,5,6,7,8,9 \ldots$ etc. so that it in fact, as necessary, grows by the first column, similarly the third column grows by the second column, the fourth by the third and so on, till reflected in the same order from the mid column stage for an arbitrary degree, whose exact $\mathrm{Y}^{\mathrm{n}}$ equation, reducing to $2 \lambda^{n}-\lambda^{n}$ in the even and $\lambda^{n}$ alone in the odd $n$ levels, can therefore always be recursively computed by the hence fully specified numerical and term suites.

These are the plain records and reckonings of the matter, served directly to any prospective opponent (or supporter) in the quasi-disputation that HHO so unjustly and unscholarly instigated - by who's order? None-the-less, the offensive gauntlet must be picked up, its diffuse libel is out in the scientific community and public alike, its arranged academic duel-field thereby
imperatively made accessible for the defence, too. To the present first riposte one must in consequence demand corresponding precision in the recharge so that this just not repeats what in the light of here summarised data clearly was devious reframing combined with overt falsification from HHO and his masters' voice in the starting round. Therefore, confronted with corpus delicti: if actual errors are there, show them! and they will be acknowledged. And if any fair and unprejudiced reader sees merit, sees the gist and truth in the positive findings, such inspiring feed-back is naturally welcome, too.

But, HHO et consortes, spare us these unworthy and substanceless innuendoes: first-line in the foregoing revealed completely hollow, second-line merely pretentious undertext allusions of lacking mathematical means and legitimacy - which for that reason alone could annul the objective results, yes, verity itself!? We feel compelled, therefore, to return in a forthcoming chapter by a demonstration that in effect we use the most original representatives and methods of genuine Diophantine whole-numbers and their operations well up to the time of Pierre de Fermat - and still going strong.

And since intelligible words failed HHO altogether at the third-line, probably most akin to Fermat, reproof stage ${ }^{2}$ but yet brought defamation into it, we will have reason to come back to that also, for which the present channel has been officially designated by HHO.s extraordinary inauguration; the more the poorer his conduct regrettably is.

## References.

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