# Vektorfelt, kortversjon

2 dimensjoner: $\mathbf{F} = \langle P, Q \rangle = P\,\mathbf{i} + Q\,\mathbf{j}$

3 dimensjoner: $\mathbf{F} = \langle P, Q, R \rangle = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$

# Vektorfelt, detaljversjon

2 dimensjoner: $\mathbf{F}(\mathbf{r}) = \langle P(x,y), Q(x,y) \rangle = P(x,y)\,\mathbf{i} + Q(x,y)\,\mathbf{j}$

3 dimensjoner: $\mathbf{F}(\mathbf{r}) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle = P(x,y,z)\,\mathbf{i} + Q(x,y,z)\,\mathbf{j} + R(x,y,z)\,\mathbf{k}$

$$\mathbf{r} = \langle x, y, z \rangle = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

# Gradient

(I tre dimensjoner – to dimensjoner er analogt):

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle = \frac{\partial}{\partial x}\,\mathbf{i} + \frac{\partial}{\partial y}\,\mathbf{j} + \frac{\partial}{\partial z}\,\mathbf{k}$$

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle = \frac{\partial f}{\partial x}\,\mathbf{i} + \frac{\partial f}{\partial y}\,\mathbf{j} + \frac{\partial f}{\partial z}\,\mathbf{k}$$

# Divergens

Om vi antar $\mathbf{F} = \langle P, Q, R \rangle = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$:

$$\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

# Curl

$$\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left( \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right)\mathbf{i} + \left( \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right)\mathbf{j} + \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\mathbf{k}$$

# Identiteter

$$\nabla \times (\nabla f) = \mathbf{0}, \quad \nabla \cdot (\nabla \times \mathbf{F}) = 0.$$