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SIF5030 Optimeringsteori
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Hjelpemidler: Godkjent kalkulator og Rottmann matematisk formelsamling
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Problem 1

Given the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$

$$f(x, y) = 5x^6 - 6x^5y + 3y^2$$

- Compute the gradient and the hessian of f . Find the stationary points of f .
- Given the starting point $x_0 = (0, 1)^t$, calculate the next iterate x_1 with the steepest descent method with exact line search.

Problem 2

We consider the following optimization problem:

$$\begin{aligned} &\text{maximize } f(x) = \mu x_1 + x_2 \\ &\text{subject to } x_1 + x_2 \leq 1 \\ &\quad \quad \quad x_1, x_2 \geq 0 \end{aligned}$$

- Plot the admissible region. Write the problem in a standard form.
- Using the simplex method, find the solutions of the problem (There are different cases depending on the value of μ).

Problem 3.

Consider the following problem:
Find the stationary points for

$$J(x) = \int_0^1 (\dot{x}^2(t) + t^2) dt$$

under the constrains

$$\int_0^1 x^2(t) dt = 2, \quad x(0) = 0, \quad x(1) = 0.$$

a) Write down the Euler equation.

b) Solve the problem

Problem 4.

Let $x : [0, 1] \rightarrow \mathbf{R}^2$, $x(t) = (x_1(t), x_2(t))$.

$$J(x) = \int_0^1 (2\dot{x}_1^2 + \dot{x}_2^2) dt.$$

a. Write down the variation of this functional, Euler equation and integrals of motion.

b. Find the curves which minimize $J(x)$ subject to

1. $x_1(0) = \sqrt{2}$, $x_2(0) = 2$, $x_1(1) = 0$, $x_2(1) = 0$
2. $x_1(0) = 0$, $x_2(0) = 0$, $x_1(1) = 0$, $x_2(1) = 2$.

c. Find the curves which minimize

1. $J(x) + 3|x(0)|^2 + |x(1)|^2$
2. $J(x) + x_2(1)^2$

Problem 5

By the Fermat principle light always moves along the path which requires the smallest time.

a Assume now that light is distributing in a media in which the speed of light depends only upon the vertical direction: $v = v(y)$. WE know that speed starts at a point with coordinates (x_0, y_0) and comes to the point with coordinates (x_1, y_1) . We want to define the trajectory of light. Write down the mathematical formulation of the problem.

b. For the problem above write down the Euler equation

c. Assume now that the media consists from three layers one above another. In the upmost layer I the speed of light is v_1 , in the next layer II the speed of light is v_2 in the next layer III the speed of light is v_3 . Let the border between the layers I and II is on the level h_1 , and the border between the layers II and III is on the level h_2 . Assume also that $y_0 > h_1 > h_2 > y_1$. Find the trajectory of light from the point (x_0, y_0) to the point (x_1, y_1) .