Zentralblatt MATH

Zbl 1191.35005

Holden, Helge; Karlsen, Kenneth H.; Lie, Knut-Andreas; Risebro, Nils Hendrik

Splitting methods for partial differential equations with rough solutions. Analysis and Matlab programs.

EMS Series of Lectures in Mathematics. Zürich: European Mathematical Society (EMS) (ISBN 978-3-03719-078-4/pbk). viii, 226 p. EUR 36.00 (2010).

The aim of this book is to be give an introduction to various types of operator splitting methods for constructing discontinuous, but physically relevant solutions of nonlinear mixed hyperbolic-parabolic PDEs. This book contains six chapters. The first describes the purpose and contents of this book. Chapter 2 demonstrates some simple examples of elementary operator splitting, and discusses the convergence of splitting approximations and briefly touches upon errors common to them. The splitting will be semi-discrete in the sense that there will he analytical solutions for the split-operators.
In Chapter 3, the authors present central elements of the mathematical framework in which to analyze, both from a mathematical and numerical point of view, second order quasilinear degenerate parabolic equations. This theory which can be considered as a generalization of the well-known Kružkov theory of entropy solutions for first-order hyperbolic conservation laws, provides the foundation for the convergence theory for operator splitting developed in the last section of the chapter.
The theory is demonstrated in Chapter 4 by applying it to one-dimensional convection-diffusion problems. The authors consider a variety of semi-discrete and fully discrete splitting methods and verify that the conditions needed to apply the abstract convergence theory hold. Moreover, the authors present several numerical examples to highlight the use of operator splitting as a basis for developing efficient numerical schemes for convection-diffusion equations. In particular, they discuss underlying error mechanisms, and in some cases suggest strategies to reduce the splitting error.
In Chapter 5, the authors extend the approach originating in Chapter 3 to yield not only convergence of operator-splitting methods but also precise error estimates, at least in the context of hyperbolic problems. They also present many numerical examples using dimensional splitting, discuss error mechanisms, and go into how to choose the splitting step to optimize runtime versus numerical errors. Chapter 6 is devoted to numerical examples for systems of equations; these systems are not covered by the rigorous analysis in the previous chapter. In particular, they discuss application from porous media flows and for two systems of conservation laws: the Euler equation of gas dynamics and the shallow-water equations.
Finally, the purpose of Appendix A is to provide the beginner with a brief introduction to numerical methods for hyperbolic problems, many of which will be used as building blocks in the splitting algorithms discussed in those chapters. *Qin Mengzhao (Beijing)*

Classification: 35-02 65M12 35L65 35K65 35A35 35M10 35Q31
Keywords: convection-diffusion equation; mixed hyperbolic-parabolic equations; porous media flow; shallow-water equations; `Matlab`
doi:10.4171/078