MR926273 (90a:81021) 81C05 (03H10 81-02 81C10 81H20)
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★**Solvable models in quantum mechanics.**

Texts and Monographs in Physics.
Springer-Verlag, *New York*, 1988. xiv+452 *pp*. \$79.00. *ISBN* 0-387-17841-4

The authors present a class of solvable models in Schrödinger quantum mechanics, in which the Hamiltonians are given by the Laplacian with point interactions (the interactions referred to in the literature also as delta-function potentials, Fermi pseudopotentials, and zero-range potentials).

To define these Hamiltonians more precisely, we let $-\Delta$ be (the negative of) the Laplacian acting in $L^2(\mathbf{R}^d)$, $d = 1, 2$, or 3, with domain consisting of smooth functions vanishing at ∞ and in a neighborhood of some fixed point x_0. It is easy to check that this domain is not a core for the operator, but that the operator has selfadjoint extensions which correspond to various boundary conditions at x_0. Such a (nontrivial) extension is referred to as a Hamiltonian with point interaction at x_0. Analogous operators can be defined with interactions at several points or even a countable number of points. If the number of points is finite, or if the points are isolated and are arranged in an infinite periodic array, it turns out that the construction of the resolvent or eigenfunctions of the operator amounts to a finite-dimensional matrix problem; in this sense, the models are solvable.

For the case of a finite number of point interactions, the authors work out the spectral and scattering theory of the operators in detail. Moreover, they show that the operators can be thought of as a suitable low energy limit of Schrödinger operators with potentials of extended support. (At low energy, the wavelength of the wave function far exceeds the support diameter of the potential.) They also analyze the case of point interactions in a periodic array, these models in two or three dimensions being analogous to the one-dimensional Kronig-Penney model.

The book includes a discussion of operators with an infinite number of randomly distributed interactions. Although certainly not solvable in the above sense, these operators provide a nice context in which to discuss the localization phenomena. There is a brief appendix which considers point interactions via nonstandard analysis.

The authors assume that the readers have a basic understanding of functional analysis, including the theory of unbounded operators. Nevertheless, there is a wealth of very pretty examples of Schrödinger operators here which could be presented, at least at a heuristic level, in an elementary quantum mechanics course.

Reviewed by *Lawrence E. Thomas*