Spectral Element Methods for Transitional Flows

Paul F. Fischer
Mathematics and Computer Science Division
Argonne National Laboratory

Abstract

We consider application of the spectral element method to simulation of transitional and weakly turbulent incompressible flows in complex domains. The spectral element method is based upon rapidly convergent, high-order weighted residual techniques employing tensor-product polynomial bases of degree N in each of K deformed quadrilateral or hexahedral elements. Because of its low numerical dissipation and dispersion, the method is well suited to simulation of flows at transitional Reynolds numbers, where physical viscosity is small. Our Galerkin formulation is stabilized at higher Reynolds numbers using a recently developed element-based filter that preserves both interelement continuity and spectral accuracy. Time advancement is based on consistent, high-order, operator splittings that permit large time steps (typ., a convective CFL of 1–5) and yield independent convective, viscous, and pressure subproblems. The latter are solved using conjugate gradient iteration with Jacobi and additive overlapping Schwarz preconditioning, respectively.

We discuss principal software components in our implementation, including matrix-matrix product kernels, a stand-alone gather-scatter utility that eases parallel software development, and a fast parallel coarse-grid solver that is central to the performance of the Schwarz preconditioner. The code scales to 1000s of processors, achieving 376 Gflops on 4096 processors of the Intel ASCI Red machine at Sandia National Laboratories.

We close with several example applications that demonstrate current capabilities, including flow in an arteriovenous graft model at Reynolds number 2700, turbulent buoyancy driven convection at Rayleigh number 10^6, and hairpin vortex formation in the wake of a roughness element for a boundary layer at Reynolds numbers 500–3500.