**A Glimm type functional for a special Jin–Xin relaxation model**

ABSTRACT We consider a special case of the Jin–Xin relaxation systems

$$u_t + v_x = 0, \qquad v_t + \lambda^2 u_x = (F(u) - v)/\epsilon. \tag{$*$}$$

We assume that the integral curves of the eigenvectors $r_i$ of $DF(u)$ are straight lines.

In this setting we prove that for every initial data $u, v$ with sufficiently small total variation the solution $(u^\epsilon, v^\epsilon)$ of $(*)$ is well defined for all $t > 0$, and its total variation satisfies a uniform bound, independent of $t, \epsilon$. Moreover, as $\epsilon$ tends to $0^+$, the solutions $(u^\epsilon, v^\epsilon)$ converge to a unique limit $(u(t), v(t))$: $u(t)$ is the unique entropic solution of the corresponding hyperbolic system $u_t + F(u)_x = 0$ and $v(t, x) = F(u(t, x))$ for all $t > 0$, a.e. real $x$.

The proofs rely on the introduction of a new functional for the solutions of $(*)$, corresponding to the Glimm interaction potential for the approaching waves of different families.